Fundamental Noise and Fundamental Constants

In this experiment a series of measurements of the Johnson noise produced by several resistors is used to derive Boltzmann's constant, and the shot noise produced by a vacuum diode are used to derive the charge of the electron. The frequency spectra of noise and the properties of noise from amplifiers are also explored.

1 *Background*

The typical use of the term "noise" in a physics experiment is to describe the limits of experimental sensitivity: a measurement is said to be "in the noise" when its mean value is about the same size, or less, than the standard deviation of that measurement. In other words, the measurement is not reproducible.

The kinds of experimental noise may be divided into two categories: Noise due to external influences such as room vibration, outside electromagnetic fields, temperature variations, and the like, we might call *interference noise.* The experimenter may be able to reduce the effects of interference noise by physically isolating the apparatus, applying electromagnetic shielding, or running the experiment at night when the building is quieter. The other category of noise comes from within the apparatus itself, and we could call it *inherent noise.* In most experiments, electronic devices make up much of the apparatus. While it is true that the noise produced by these devices depends on their design and operation, some of the noise is a result of fundamental physical processes and quantities such as the ambient temperature, circuit resistance, and the discrete nature of electric charge. Noise of this type we may call *fundamental noise.*

Two types of fundamental noise will be studied: thermal, or "Johnson", noise which is present in any kind of conductor and depends only on the conductor's resistance and temperature, and "shot" noise which is mainly evident in devices where the charge carriers overcome a potential barrier, such as a diode. In shot noise, the noise is due to fluctuations in the electric current which come from the fact that each charge crosses the barrier independently of the other charges; the variation in the current depends on the statistical variation in arrival times of the charge carriers times the charge on each carrier.

Both Johnson noise and shot noise share the interesting feature that the amplitude of the energy fluctuations they produce is *independent of frequency.* In other words, if you were to measure the RMS power of the noise in the ranges 100 to 200 hertz, 1100 to 1200 hertz and 10,100 to 10,200 hertz, you would get the same answer in all three cases. This kind of noise is called "white". The term comes from the analogy with light; white light is the light you get when all colors (all frequencies) are present in equal intensity. Since RMS power is proportional to the square of RMS voltage, in white noise the RMS amplitude of voltage fluctuations varies as the *square root* of the frequency band.

Not all noise is white. Interference noise is frequently confined to a few frequencies, the most common example being the noise from power-line interference which shows up at 60 Hz and its harmonics (120 Hz, 180 Hz, etc.). Another kind of noise spread over many frequencies ("broadband noise") is called "$1/f$ noise", since the noise power is varies inversely with frequency. The physical origin of $1/f$ noise is somewhat mysterious, as $1/f$ fluctuations show up in a wide variety of physical systems, for example, variations in the resistance of resistors, ocean current velocity variations, pitch and loudness variations in music, and in the frequency versus intensity of earthquakes. See the paper by Milotti [1] for a review of this ubiquitous kind of noise.

Given the white spectrum of Johnson and shot noise, one must take the frequency response of the measuring system into careful consideration whenever making measurements of this kind of noise. In this experiment, our task is greatly eased by the use of a sophisticated piece of gear—an FFT spectrum analyzer. This device measures an electrical signal over some time interval Δt and then performs a Fast Fourier Transform on the data to display directly to the user a graph of the RMS voltage versus frequency. By using the analyzer, one can see immediately that the spectra of Johnson and shot noise are indeed white; moreover the experimenter can make precise measurements of the average RMS voltage over a well-defined band of frequencies.

Because the origin of Johnson and shot noise derives from fundamental physics and because this physics involves fundamental constants, one can, in fact, use measurements of noise to obtain experimental values of these constants. Let us look more closely at the theoretical results of Nyquist and Schottky, the physicists who derived the theorems stating the relationships of Johnson and shot noise.

Johnson noise, named after J. B. Johnson, the first experimenter to verify the effect, is due to *thermal* fluctuations within a conductor of resistance R and temperature T; it is a consequence of the same kind of physics which gives rise to blackbody radiation from hot objects. Nyquist showed that Johnson noise obeys the following formula:

$$V_{\mathrm{Johnson}} = \sqrt{4kTRB}\ , \tag{1}$$

where k is Boltzmann's constant (1.38×10^{-23} J/K), T is the kelvin temperature of the resistor, R is the resistance (ohms), and B is the bandwidth of the measurement in hertz. The voltage V_{Johnson} is the root-mean-square (RMS) voltage that a noiseless RMS voltmeter with a bandwidth B would measure by simply connecting it across a resistor of resistance R which is at temperature T. A derivation of equation (1) can be found in the book by Kittel and Kroemer [2, pp. 98–102]. However, it is fascinating to read Nyquist's original derivation published in 1928; he gives an elegant and concise argument based on the necessary requirements of the 2nd law of thermodynamics and the physics of electromagnetic waves. The paper is available online [3], along with Johnson's original measurements [4], which verified that the noise does not depend on the type of resistor used.

The name "shot noise" comes from the German word *Schrotteffekt* or "shot effect" used by W. Schottky, who first gave it a thorough theoretical explanation. Schottky described the noise as being like the sound of a handful of lead shot poured slowly onto a steel plate. It is the same noise one hears when rain hits a window, or from a Geiger counter held near a radioactive source.

As noted above, shot noise is related to the discrete nature of electrical charge. If you were able to count the number of electrons $n(\Delta t)$ crossing from the cathode to the anode of a vacuum diode during a succession of short time intervals of size Δt, you would find that this number would be distributed about a mean value $\bar{n}$. The RMS fluctuations in n, Δn_{RMS}, would give rise to fluctuations in the current, or "current noise". We define the current noise as the RMS fluctuation in the current as follows:

$$I_{\mathrm{shot}} \equiv \left(\frac{1}{N} \sum_{i=1}^{N} \left(I_i(\Delta t) - \bar{I} \right)^2 \right)^{1/2} , \tag{2}$$

where i labels the N measurements made, $I_i(\Delta t) = e n_i(\Delta t)/\Delta t$, and $\bar{I} = e\bar{n}/\Delta t$. Simple substitutions show that I_{shot} is proportional to Δn_{RMS}, and these are known to obey Poisson statistics.

From the theory of Poisson statistics, we have $\Delta n_{\text{RMS}} = \sqrt{\bar{n}}$. The time interval of measurement Δt is related to the bandwidth B of the measuring device by $B = 1/(2\Delta t)$ (another theorem due to Nyquist). Thus,

$$I_{\text{shot}} = \sqrt{2e\bar{I}B}\ . \tag{3}$$

In this experiment the mean current $\bar{I}$ is the "emission current" passing through the vacuum diode. This current, I_{em}, is given by the electrons emitted by the cathode and collected by the anode of the diode.

Johnson noise is measured by placing a resistor directly across the input terminals of an amplifier and then measuring the amplified output voltage. Shot noise is measured by creating an electron emission current in a vacuum diode and then passing this current, collected at the anode, through a resistor. The voltage across the resistor is applied to the amplifier. See the circuit diagram of our shot noise box in Fig. 2.

It is instructive to calculate the magnitude of the voltages generated in this experiment. For resistances of several thousand ohms and a bandwidth of $\approx 10^4$ Hz, the Johnson noise is on the order of a few microvolts, and for diode emission currents of $\approx 10\ \mu$A, the shot noise voltage developed across the anode resistor is also on the order of microvolts. Microvolt signals are small by the measurement capabilities of most laboratory voltmeters, and for this reason a quiet amplifier is used to bring the signal up to a more easily measured level.

Exercise 1 *Verify the approximate results for the magnitude of the noise signals mentioned in the previous paragraph. Calculate the Johnson noise produced by a 10kΩ resistor at 300 K (room temperature) and at 77 K (liquid nitrogen) if measured with a voltmeter of 10 kHz bandwidth. Repeat the calculation for shot noise, assuming a 10 μA emission current.*

The amplifier used in this experiment is a very good one, but because it is not ideal, it makes its own contribution to the noise voltage measured at its output. One manifestation of the amplifier contribution is the non-zero output of the amplifier even when the input is shorted ($R = 0$). However, this contribution is present even when $R \neq 0$, and indeed, varies a bit as a function of R. Additionally, the RMS noise of the amplifier typically scales with the gain of the amplifier. Engineers usually account for this fact by treating the noise at the output as if it were due to a noise signal present at the input of an otherwise noiseless amplifier. So amplifier noise specifications are given in terms of "equivalent input noise". See the text by Horowitz and Hill for a thorough and accessible discussion of amplifier noise [5, pp. 428–449].

How then, should the amplifier contribution be subtracted from the noise voltage measured at the amplifier output for a given R across the input terminals?

Before addressing this question, let us consider how two noise voltages (in this case the Johnson noise across the resistor and the noise contribution of the amplifier) are combined. Both of these noise voltages consist of many component frequencies with random phases and amplitudes; the average value over time for each of these signals is zero. Hence, the RMS value of the sum of the two voltages is not simply the sum of the RMS values of each one, as can be seen by a direct calculation. Let V_1 and V_2 be the instantaneous values of the two noise voltages. Then the mean square value is

$$\overline{(V_1 + V_2)^2} = \overline{V_1^2 + V_2^2 + 2V_1V_2}\ . \tag{4}$$

Since the average value of a sum of quantities is the sum of the averages of each, and since the phases and amplitudes of V_1 and V_2 are random, the average $\overline{V_1V_2}$ equals zero, but $\overline{V_1^2}$ and $\overline{V_2^2}$ do not. Thus,

$$\overline{(V_1 + V_2)^2} = \overline{V_1^2} + \overline{V_2^2}. \tag{5}$$

When V_1 and V_2 are combined this way, they are said to be added in "quadrature."

As an example of the use of quadrature summing, the voltage measured at the amplifier *output* can be expressed as the quadrature sum of the Johnson noise voltage and the amplifier contribution (now with the RMS averaging assumed to be inherent in the measurements):

$$V^2_{\text{measured}} = G^2 V^2_{\text{Johnson}} + V^2_{\text{amplifier}} = G^2 4kTRB + G^2 v^2_{ein} \ , \tag{6}$$

where G is the amplifier gain and v_{ein} is the equivalent input noise of the amplifier. Note that spectrum of v_{ein} is not necessarily white (although it often is nearly so, over a limited band of frequency); however, the signal is still random. If the amplifier noise does not depend noticeably on R, one can obtain $V^2_{\text{amplifier}}$ by making a measurement with $R = 0$. To account for R dependence of v_{ein}, one possibility is to hold R constant and take measurements at two different temperatures. This can conveniently be done at room temperature, typically about 295 K, and at liquid nitrogen (LN2) temperature, 77 K, yielding

$$V^2_{\text{measured,295K}} - V^2_{\text{measured,77K}} = G^2 4kRB(295\text{K} - 77\text{K}) = G^2 4kRB(218\text{K}) \tag{7}$$

Measurements at the two temperatures will thus allow us to subtract out the amplifier contribution to the Johnson noise measurement.

The problem of amplifier noise is handled more simply with the shot noise measurements. As mentioned earlier, the Johnson noise contribution of the anode resistor and also the amplifier contribution need to be subtracted from the measured shot noise signal. But these two contributions are just the measured signal when the emission current $I_{\text{em}} = 0$, where in this case R is always the same, equal to the anode resistance.

You may be interested in how other labs have implemented similar experiments. References [6] and [7] are worth a look. These and other examples and information are available on the class website.

2 *Procedure*

The measurements for this experiment will be taken with a Stanford Research Systems SR760 FFT Spectrum Analyzer. This device has a number of capabilities, of which we will use only a few. The analyzer produces a spectrum—a graph of voltage versus frequency—by recording a sequence of voltage measurements (like a digital oscilloscope) and then performing a Fast Fourier Transform on the measurements (hence the initials "FFT"). The results of the Fourier transform can be manipulated in various ways. For this experiment we will use two of these manipulation features to obtain the noise voltages:

Spectrum Averaging. Since the noise we are interested in is random over a wide frequency range, successive FFT spectra will show significant variations from one spectrum to the next. The SR760 allows the user to average successive spectra in order to reduce these variations. For this experiment, you will want to set up the averaging feature to make "exponential" RMS averaging of 100 spectra.

Band Analysis. In the course of FFT analysis, the frequency axis, or "span" is divided into 400 equal intervals called "bins". For a full-width spectrum of 100 kHz, each bin represents an interval of 250 Hz; if the span is narrowed, each bin is correspondingly narrowed. With band analysis, one can combine the measurements in adjacent bins to give the average voltage over a frequency interval larger than the width of a single bin. We will use this feature to give the RMS voltage of a band of 1000 Hz width which is centered on 10 kHz: the RMS sum of all of the (averaged) bins between 9,500 Hz and 10,500 Hz.

To start, set up the SR760 to average 100 spectra and analyze a 1000 Hz band centered at 10 kHz. You may figure this out for yourself by reading the relevant pages of the manual, or follow these specific steps. Note: in these instructions, [LABEL] means a real button labeled 'LABEL', but softkeys—unlabeled buttons next to the analyzer screen—will be denoted by double quotes: "Label".

1. Turn on the SR760 to its standard default state by pressing the power button (at the lower left corner of the instrument panel) while simultaneously holding down the backspace button, [←], until you see the self-test symbols appear. Wait for the "Calibrating Offset" window to disappear.

2. *Set up spectrum averaging.* Press [AVERAGE], then use the softkeys to turn "Averaging" on, "Average Type" to RMS, and "Average Mode" to exponential. Finally press the softkey "Number Averages" and use the numbered keypad to set the number of averages to 100: [1] [0] [0] followed by the softkey "Enter". Press [START] to start the spectrum averaging, and note how the spectrum becomes much smoother.

3. *Choose useful units and display type.* Press [MEAS] to call up the Measure Menu. Select "Measure" and then either "Spectrum" or "PSD", your choice. Note that PSD displays the power spectral density, which is the magnitude of the spectrum normalized to a 1 Hz bandwidth, so it displays readings in volts/$\sqrt{\text{Hz}}$. Press "Return" and then select "Display" followed by "Log Mag.". Press "Return" again, select "Units" followed by "Volts RMS". End this step by pressing "Return".

4. *Set up band analysis.* Press [ANALYZE] followed by the softkey "Band". Press "Band Center", and enter 10 kHz from the keypad ([1] [0] and then the "kHz" softkey). Then press "Band Width" and enter 1 kHz for this parameter (you know how now!). End this step by pressing "Return".

Turn on the analog oscilloscope. The scope is useful to be able to see the noise signal in real time.

Make sure that the output of Low-noise amplifier is connected to the A input of the SR760 and oscilloscope CH1 input, and turn on the ±15V power supply to the amplifier and shot-noise box (which will be used later).

CAUTION: The low-noise amp is a sensitive instrument. *PLEASE* be kind to it. Before connecting or disconnecting any sources or BNC cables to the input, always switch the input to ground.

Make sure that the input of the low-noise amp is set to "GND". Then press [AUTO RANGE] to set the input of the FFT analyzer to an appropriate gain, followed by [AUTO SCALE] to select an appropriate vertical scale to display all of the data.

2.1 Johnson Noise Measurements

The Johnson noise measurements are easier to take, so it is a good idea to start with them and then move on to the shot noise measurements. There are 8 resistors for the Johnson noise measurements (9 including 0Ω). They are attached to the cables with the copper tubing tip: the resistor in each cable has one end soldered to the center conductor of the cable and the other clamped by and soldered to the pinched end of the copper tubing. The other end of the tubing is soldered to the cable shield. Thus, the resistor is completely enclosed by the tubing shield, which minimizes electromagnetic interference. The resistors are of the metal-film type, whose resistance value changes very little ($< 1\%$) between room temperature and 77 K.

Caution: Any contact of liquid nitrogen and bare skin can cause severe injury. Please handle the liquid nitrogen very carefully.

Fill the dewar to approximately half-full with LN2. It will take a while for the vigorous boiling to stop, and after it does you will need to add some more LN2 to bring it back up to the approximately half-full level .

Do a quick check to make sure the Johnson noise measurements will work properly, as follows. Make sure the input on the low-noise amp is set to "GND" and plug the $40\text{k}\Omega$ resistor into it. (Note: the connectors are very tight; make sure you grip the bayonet ring firmly when you make or break the connection.) Then switch the input select to "Input BNC", and watch the spectrum level rise on the SR760. Press the [SCALE] button and use the softkeys "Top Ref." and "Y/Div" to arrange the display so that the level changes a substantial amount when you switch the low-noise amp input select between "GND" and "Input BNC". (Typical values are Top Ref. = 25 mV, Y/Div = 5 dB; you can use [EXP] [−] [3] to enter millivolt values.)

If all looks good, you will want to reduce the frequency span on the FFT analyzer in order to improve the accuracy of the band analysis measurement. First, set the low-frequency limit to 0 Hz by pressing [FREQ], "Start Freq.", [0], and then "Hz". Then, press [SPAN▽] repeatedly in order to bring the span down to 12.5 kHz. You should see a display something like that shown in Fig. 1.

Here are a few hints on running the SR760 which you may find helpful as you do this experiment:

- Press [START] to start (or restart) a new spectrum analysis. Press [PAUSE CONT] to stop a measurement and freeze all of the data. You can continue with the same measurement and averaging by pressing [PAUSE CONT] again.
- One can use the marker to look at the amplitude of specific frequency components. Press [MARKER] to activate control of the marker, and use the knob to change its position.
- The occasional automatic offset calibration can be disabled by pressing [INPUT] followed by setting the softkey "Auto Offset" to off.
- The key-press beep can be disabled by pressing [SYSTEM SETUP], "Setup Sound" and then setting "Key Click" to off.
- If you would like a printout of the display screen, first check that the printer type is set to HP (Hewlett-Packard Laserjet) by going to the [SYSTEM SETUP] menu, selecting "Setup Printer" and toggling "Printer Type" to HP. Then when you press the [PRINT] button, the screen will be printed on the attached printer.

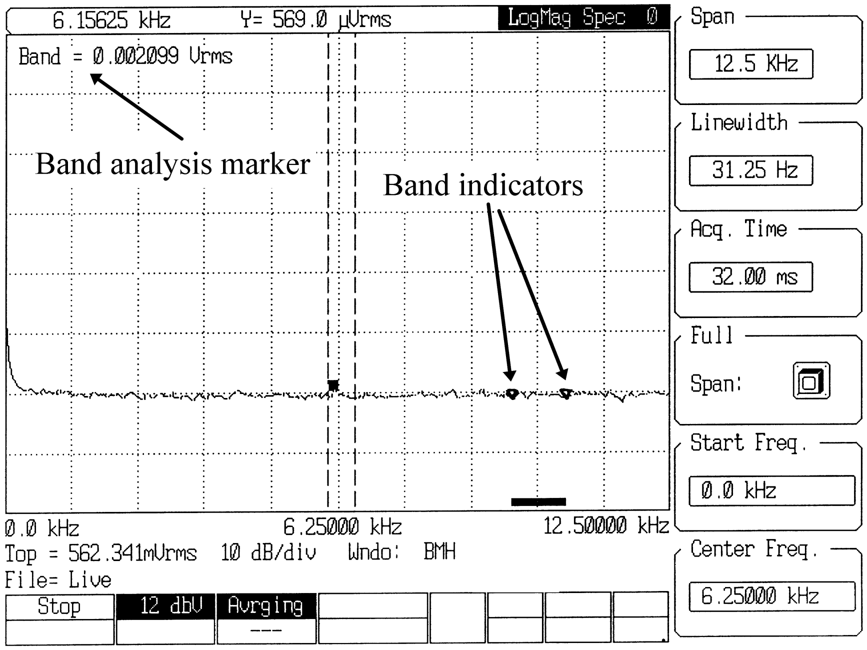

Figure 1: Display screen of the SR760 analyzer. Note the band analysis indicator at the top left hand corner and the inverted triangle indicators on the data trace denoting the limits of the band. The information shown in the right column can be obtained by pressing [FREQ].

Once you are ready, follow these steps to collect noise data for ***each resistor, one at a time with both temperatures, starting with the 40kΩ resistor, and proceeding in order down to 0Ω:***

1. Set the input on the low noise amp to "GND".
2. Plug a resistor cable into the input and turn the input select to "Input BNC". (Grip the connector barrel firmly!)
3. Watch the spectrum on the SR760 rise and settle. Note the reading of the "`Band =` " indicator. This gives the average RMS reading of the voltage over that band. Press [PAUSE CONT] to stop the reading, and record the voltage.
4. Since the band reading fluctuates, you should take a few readings and average the result. Use the [PAUSE CONT] button to take a total of 5 (or more) readings, allowing a bit of time to pass between each. (Alternately, you could set the number of spectrum averages to 500 or 1000, and the averaging mode to "Linear", and just take one reading, but you will need to restart the averaging/reading at the end of each averaging cycle.)
5. After collecting room temperature measurements for that resistor, plunge the end of the cable into liquid nitrogen. It is interesting to watch the spectrum and the scope screen as the resistor cools. You will see the spectrum jump as low frequency transients from "microphonics" and thermal contractions appear. Additionally the spectrum level will fall. Wait until the bath and signal settle—a couple of minutes, then repeat the 5 or so measurements for the 77K temperature.

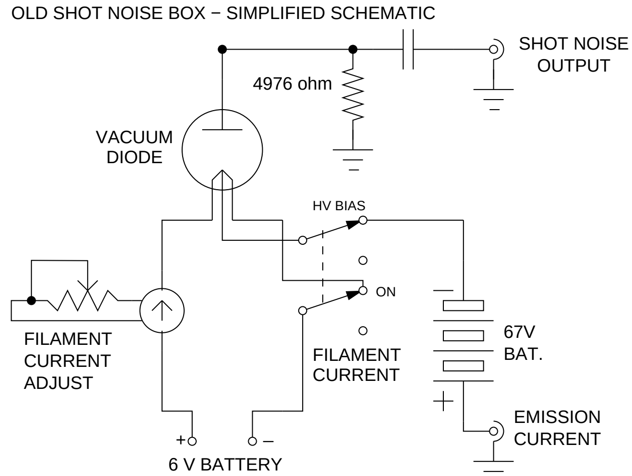

Figure 2: Simplified shot-noise source circuit. The noise is produced by the emission current of the vacuum diode. (This diagram is for the old noise source.)

6. Repeat the above steps for the each resistor in turn, finishing with the 0Ω cable.

Analysis of the Johnson noise measurements is discussed later in this write-up. You may save time, however, if a member of your group enters the readings into a spreadsheet (KaleidaGraph or Excel) as you go.

2.2 Shot Noise Measurements (Old shot noise source)

The shot noise is produced by the "Shot Noise Source" whose circuit is shown in the diagram of Fig. 2. The primary element of the box is the vacuum diode,which is inside the metal box. A vacuum diode (like all vacuum tubes) works by thermionic emission, which is the name given to the process by which electrons come off of a heated wire under the influence of an electric field. In this case, the wire is heated by the "filament current" which is produced by a current source. The current source is adjustable with a screwdriver by means of a small 10-turn potentiometer. The electric field in the vacuum diode is created by a 67V battery inside of the box whose negative terminal is connected to the cathode of the vacuum diode and whose positive terminal is connected to ground through the DMM ammeter which is attached to the "Emission Current" connector. The anode side of the diode is connected to ground through a 4876Ω resistor. Thus the cathode is at a negative potential relative to ground, and electrons whose energy is raised by the heating from the filament can escape the cathode and move toward the anode. The current of electrons flows through the anode resistor. Fluctuations in the current—the shot noise—are seen as fluctuations in the voltage drop across this resistor. The anode resistor has its own Johnson noise contribution to the measured voltage, and this contribution, along with others from the internal electronics, needs to be subtracted out (by quadrature) to isolate the shot noise contribution.

Make sure that the "FILAMENT CURRENT" switch is off.

Connect the battery to the red and black wires. Red to the POSITIVE (+) terminal, and black to the NEGATIVE (−) terminal. Do NOT connect any part of the battery to ground. The 6V battery is itself held at 67V below ground by the internal bias battery, so grouding one of the terminals will short out this bias.

Move the low noise amp (with the input switched to "GND") over to the shot noise source and connect its input to the "Shot Noise Output" via the BNC adapters. (We use these adapters rather than a longer cable to reduce the external noise pickup.)

Check that a Keithly model 2000 DMM is connected to the emission current measurement connector "I_{em} output", and is configured to measure DC current. (If not, make sure that the connector is connected between the white "AMPS" and black common terminals, and press the "DCI" button to enable the current measurement.)

Use the small screwdriver to turn the "FILAMENT CURRENT ADJUST" fully counterclockwise. (if you feel a small click as you turn the screw, you are there.)

Now, turn the "FILAMENT CURRENT" switch ON, and with the DMM at its most sensitive setting, wait a few tens of seconds to see that the emission current starts to rise. It could rise as much as 150 microamps (0.150 mA), but should only come up to about 0.010-0.040 mA. If it is more than this, try adjusting the screw counterclockwise, and see if the current drops.

Turn the "FILAMENT CURRENT ADJUST" screw clockwise and note the change on the DMM. You should see the current begin to increase. See if you can set the current to 50.0 microamps, and then 100.0 microamps.

Then proceed to take shot noise data as follows:

1. With the low-noise amp input set to "GND", set the Emission current to about 10.0 microamps (0.0100 mA).

2. Set the spectrum analyzer to measure the full 100 kHz spectrum. (Press FREQ, and push the "Span:Full" softkey) Turn the low-noise amp input switch to "Input BNC" and watch the spectrum change. You should see a largely "white" (i.e., flat) spectrum after the transients die down. You will also see a number of "spikes" on the spectrum corresponding to noise from the outside environment. Fortunately at the 10 kHz frequency, the outside interference is minimal. Notice how the overall noise floor increases as you turn the "FILAMENT CURRENT ADJUST" knob up. Set it all the way to its clockwise maximum, and see that the emission current increases to a little over 150 microamps (0.150 mA). Also note that the noise floor overtakes most of the outside interference spikes.

3. Now, set the analyzer as in the Johnson noise measurements: look at a 12.5 kHz span, and set the band analysis to measure the RMS voltage of a 1 kHz band centered at 10 kHz.

4. Turn the emission current knob down to 10 microamps (0.010 mA), ad let the current settle a bit.

5. Take measurements with the spectrum analyzer in the same manner as for the Johnson noise: 5 readings with exponential average of 100 spectra, or one linear-average reading of 1000 measurements. Watch the emission current in case it drifts from the setting.

6. Repeat the measurements for emission currents spaced evenly between 0.010 mA and 0.150 mA, so that you have between 10 and 20 data points for emission current versus noise voltage.

7. Finish by disconnecting the 6V battery. This will kill the emission current, but leave all other electronics powered up. This final measurement will give the noise due to the combined effects of the Johnson noise of internal resistor (4976 Ω), the noise contributed by the regulating circuit, the noise of the low-noise amp, and the noise of the analyzer's input amp.

3 Data Analysis

The data for the Johnson noise and shot noise are analyzed similarly. First, you must subtract out the "zeroth order" noise from the measurements by following the prescription discussed in Section 1, where we noted that uncorrelated signals add (and subtract) in quadrature. Second, you need to then scale the results by the gain of the low-noise amp, and in the case of the shot noise, convert the resulting voltage measurements to current measurements with Ohm's law. Finally, you may apply the theoretical equations (1) and (2) to obtain values for the fundamental constants k and e. The specifics of the analysis, and the requirements for your report are given in the following exercises.

Exercise 2 *Enter your Johnson noise data into a spreadsheet or KaleidaGraph worksheet and plot the results. Note and correct any obviously anomalous values. Then reduce the data by subtracting, via quadrature, the* 0Ω *noise and confirm that the noise follows the Johnson noise formula. Don't forget about the gain of the amplifier! The bandwidth* $B = 1000$ *Hz, as was set on the analyzer. Produce plots of* V^2_{Johnson} *versus* R *for the two temperatures.*

Fit your plots with a line, then calculate k *for both the room-temperature and 77K readings from the slope of the lines. Also calculate* k *by taking the difference between the two sets of readings, as per the discussion in Section 1, which minimizes the effects of a changing source resistance on the amplifier noise.*

Compare your values for k *with the accepted value. Make sure to include and propagate the error in your measurement so that your comparison makes sense.*

A note on error analysis. You can assume that the temperature is known to ± 1 K and that the bandwidth B is known to the width of one FFT bin, which would be the span (in Hz) divided by the number of bins, which is 400. (Example: for the full span of 100 kHz, the bin width would be 250 Hz, but you reduced the span to 12.5 kHz, right?) The gain of the amplifier has been measured and found to be within 3% of the nominal value. The other uncertainties can be taken from the fitting error, correctly scaled.

Exercise 3 *Use your results for room temperature to calculate the noise figure of the low-noise amplifier in the frequency range studied. The noise figure* NF *is defined as*

$$NF = 20 \log_{10} \frac{V_{\text{measured}}(R)}{\text{Gain} \times \sqrt{4kTRB}} \text{ dB} \tag{8}$$

In other words, it is the decibel difference between the noise from a real amp with a particular resistance R *at the input and an ideal noise-free amp with that same resistance* R *at its input. A very good amplifier will have noise figures below 1 decibel. How does this amp compare?*

Exercise 4 *Analyze the shot noise data by first converting your RMS voltage measurements into RMS currents (use Ohm's law, of course, and don't forget about the amplifier gain). Make sure to subtract (properly) the zero current noise from the data. Make a plot of* I^2_{shot} *versus* I_{em}*, and fit the result to a line.*

Use the fit result, along with the bandwidth of your measurements, to find a value of the electron charge e *from the shot noise formula. Calculate the error in your measurement, and compare your result to the known value of* 1.602×10^{-19} *C.*

4 Optional Exercises

These exercises explore some other aspects of spectra and noise. The first one uses a $1/f$ noise source. This noise is produced by applying a weak *forward* bias on a 4.7 volt zener diode, not the usual reverse bias normally employed with zeners.

The second exercise uses the capability of the SR760 to measure the amplitudes of specific frequencies and their harmonics.

Exercise 5 *The "$1/f$ Noise Generator" produces a noise power spectrum that follows, roughly, the rule*

$$P_{\text{noise}} \propto V_{\text{noise}}^2 \propto \frac{1}{f^{\alpha}}\ , \tag{9}$$

where the exponent α lies somewhere in the neighborhood of 1. Use the low-noise amp and SR760 to measure the power spectral density (PSD) over a range of frequencies with this source. Also take measurements with the low-noise amp input set to "GND". Use this data to determine the proportionality constant and the value of the exponent α.

Exercise 6 *Use a function generator to produce waveforms of a specific type (sine, square, triangle, etc.). Feed the signal into the SR760. Work through the "Getting Started" exercises in the SR760's manual to figure out how to perform harmonic analysis. By these analyses see whether the Fourier series representations of the various waveform types conform to the measurements. For example, a square wave of frequency f can be decomposed into terms at f, $3f$, $5f$, ..., with relative amplitudes of 1, 1/3, 1/5, ... The Fourier decompositions of other waveforms can be found in mathematical handbooks.*

References

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