PHYS 536

R. J. Wilkes

We'll start class at 7:05

Session 2

Harmonic oscillators:

Simple, damped, driven

resonance

1/5/2023

Course syllabus and schedule – first part…

See : http://courses.washington.edu/phys536/syllabus.htm

Session	date	Day	Readings:	K=Kinsler, H=Heller	Topic
1	3-Jan	Tue	K ch. 1	H: Ch. 1, 2	Course intro, acoustics topics, overview of wave properties; pulses, transverse and longitudinal waves, overview of sound speeds
2	5-Jan	Thu	K ch. 1	H: Ch. 9, 10	harmonic oscillators: simple, damped, driven; complex exponential solutions, electrical circuit analogy, resonance, Q factor ← Tonight
3	10-Jan	Tue	K ch. 1	H: Ch. 3	Fourier methods: Fourier series, integrals, Fourier transforms, discrete FTs, sampling and aliasing
4	12-Jan	Thu	K. chs 10	H: Ch. 4, 11	Frequencies and aliasing; convolution and correlation; discrete convolution; digital filtering, optimal filters, FIR filters, noise spectra; power spectra. REPORT 1 PROPOSED TOPIC DUE
5	17-Jan	Tue	K. ch. 2, 3, 4	H: Ch. 13, 15	waves in strings, bars and membranes; Acoustic wave equation; speed of sound; Harmonic plane waves, intensity, impedance.
6	19-Jan	Thu	K. Ch. 5, 6	H: Ch. 1	Spherical waves; transmission and reflection at interfaces
7	24-Jan	Tue	K. Ch. 8	H: Ch. 7	Radiation from small sources; Baffled simple source, piston radiation, pulsating sphere;
8	26-Jan	Thu	K: Ch. 10	H: Chs. 13-15	Near field, far field; Radiation impedance; resonators, filters
9	31-Jan	Tue	K. Ch. 9-10	H: Chs. 16-19	Musical instruments: wind, string, percussion
10	2-Feb	Thu	K. Ch 14		Transducers for use in air: Microphones and loudspeakers
11	7-Feb	Tue	K. Ch 11	H: Chs. 21-22	The ear, hearing and detection
12	9-Feb	Thu	K. Chs 5,11		Decibels, sound level, dB examples, acoustic intensity; noise, detection thresholds. REPORT 1 PAPER DUE by 7 PM; REPORT 2 PROPOSED TOPIC DUE

From last time:

Course information

- T-Th, 7:00-8:50 pm, A110 Physics-Astronomy, and/or Zoom
  - **I will attend by Zoom only for most sessions**
  - In-person (optional) meetings on a few later dates for demonstrations
- R. J. Wilkes
  - 206-543-4232 , wilkes@uw.edu
    Office hrs: after class, or by appointment via zoom/phone
- Website: http://courses.washington.edu/phys536
- Books:
  - *Fundamentals of Acoustics* 3rd ed. (1982) or 4th ed. (2000), Lawrence E. Kinsler, A. R. Frey, A. B. Coppens, J. V. Sanders; Wiley
  - *Why you hear what you hear*, Eric J. Heller, Princeton University Press, 2013, ISBN: 978-0691148595
  - Books are on 2 hr reserve in the UW Odegaard Undergraduate Library – see http://www.lib.washington.edu/about/hours/
- Grades – see class calendar on website for due dates
  - Two 5 p term papers (100 points each) - submit proposed topics by email
  - End-of-term exam (100 pts, take-home; *no* final exam in exam week)

Announcements

- SPECIAL: Next Tuesday, class will start at 7:30 pm, not 7 pm.
  - Most sessions thereafter will be Zoom only (I will not come in)
    - HOWEVER: you are quite welcome to come to A110 PAB to meet with fellow students and attend the zoom sessions together
    - Later, I will come in and some sessions in-person, when demonstrations are part of the show – I'll notify you a week in advance of these dates
      - You can watch me do the demos on Zoom, of course

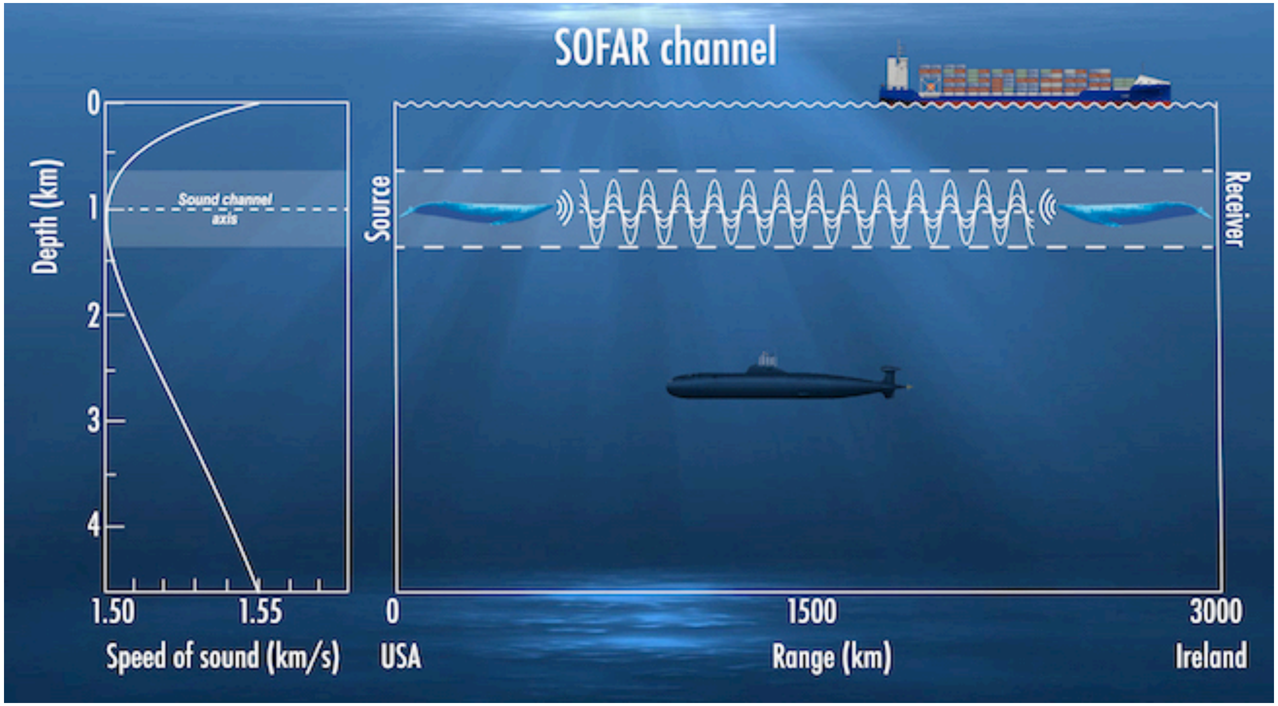
Questions from a student last time

Bubba C. figured out the answers for himself later, but for benefit of everyone else:

- In air, the density will be smaller when temp is higher, so why is the speed of sound faster in hotter air? Or is the speed of sound faster in hotter air only if the density is constant between the hot and the cold air?
  - The plots shown were for density and other parameters held constant
- For the SOFAR layer, I can see why there would be total internal reflection at 1000 m below the surface, but why would there be reflection at 100 m below the surface?

– The channel is centered on the sound speed minimum and reflection occurs because the sound speed increases both above and below the minimum around 1000 m – compare to index of refraction in optics.

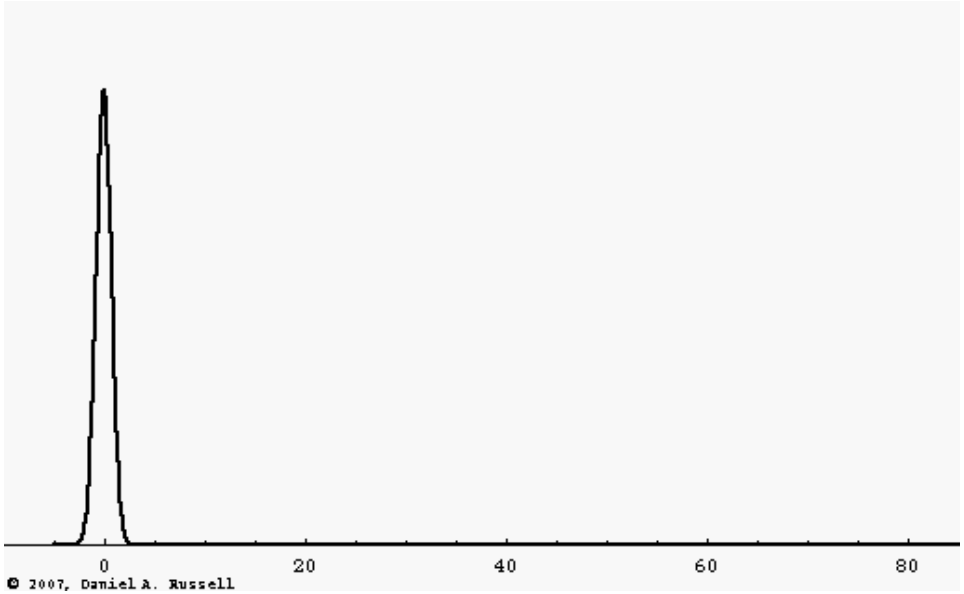
PS: Thanks for the nice picture



Sound speed and frequency

- Speed of sound does not vary much with *f*
  - If *c* depended on *f,* sound signals would change significantly depending upon how far away you are
    - This is called "dispersion"
  - Small *f* dependence can be observed, for example in undersea sound transmission
    - A pulse with many frequencies in it will spread out in time as it travels
    - Pitch will vary – pulse spreads; ping becomes a "chirp"

Pulse moving in a dispersive medium

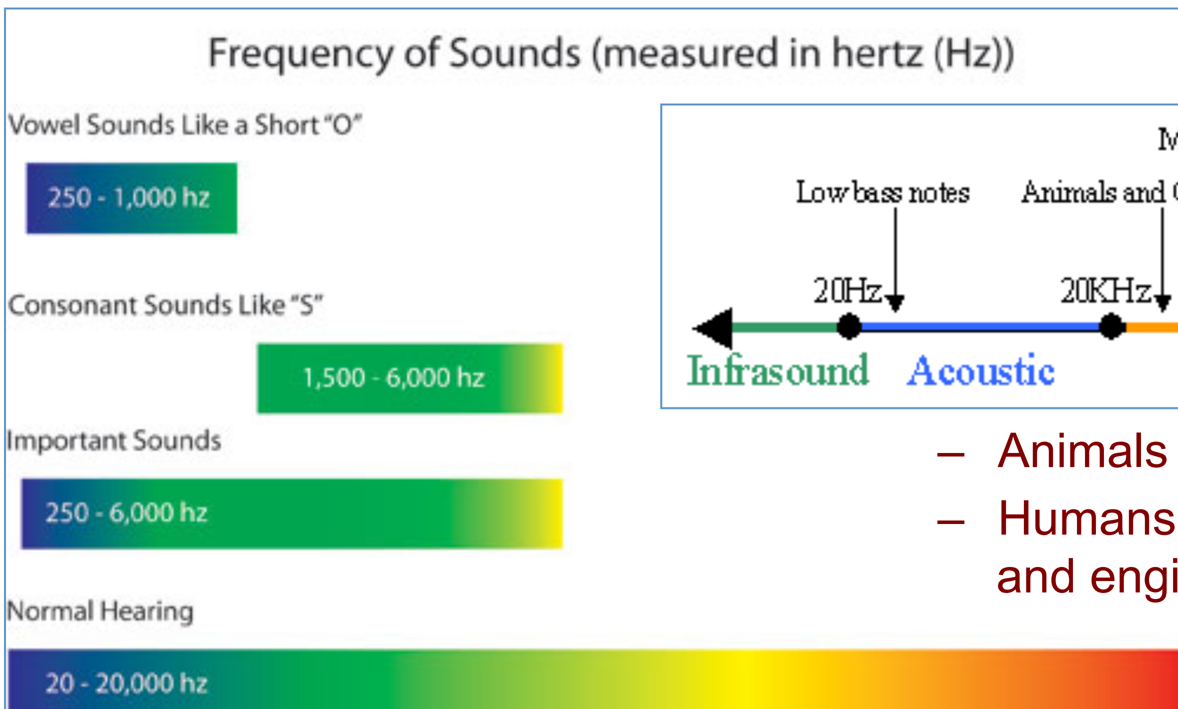


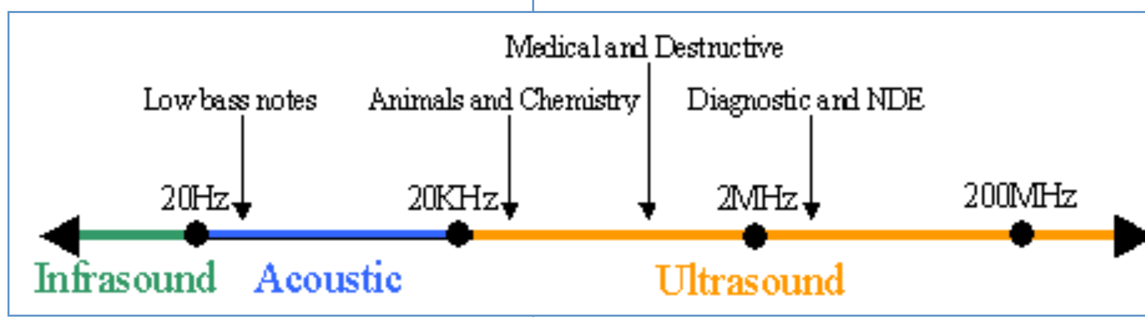
Frequency range terminology

- Audible/"Acoustic" – nominally 20 Hz to 20 kHz

  (actual range for most people is closer to 50Hz-15kHz)
- Infrasonic - below audible

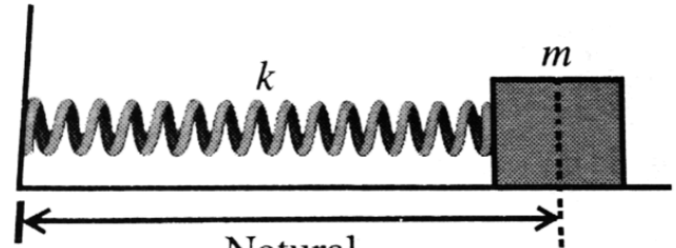
  (below about 0.1 Hz we call it "**vibration**" !)
- Ultrasonic - Above 20 kHz





- Animals use infra/ultra ranges
- Humans use ultrasound in medicine and engineering/science

Harmonic oscillators



- First and easiest case: simple harmonic motion

$$f = -sx = m\frac{d^2x}{dt^2} \quad \rightarrow \quad \frac{d^2x}{dt^2} + \frac{s}{m}x = 0$$

Using Kinsler's notation: spring constant k = *s*

"neglect friction"

$$\text{with } s, m \text{ both} > 0, \quad \text{let} \quad \omega_0 = \sqrt{s/m} \quad \rightarrow \quad \frac{d^2x}{dt^2} + \omega_0^2 x = 0$$

Solutions come in three forms:

$$I: \quad x = A_1\cos\omega_0 t + A_2\sin\omega_0 t \quad \text{where} \quad \omega_0 = \text{angular frequency (radians / sec)}$$

$$f_0 = \frac{\omega_0}{2\pi} \quad (\text{'linear'}) \text{ frequency (cycles / sec} = Hz)$$

$$\text{Period}: \quad T_0 = 1/f_0 \text{ (time for 1 cycle)}$$

$$II: \quad x = A\cos(\omega_0 t + \phi_0)$$

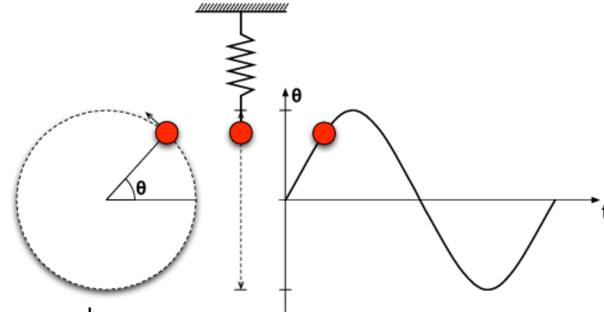
$$\text{where} \quad \phi_0 = \text{initial phase angle (phase at } t = 0)$$

$$III: \quad x = Ae^{\gamma t}, \quad \text{where} \quad \gamma^2 = -\omega_0^2 \quad \rightarrow \quad \gamma = \pm i\omega_0$$

$$\text{So} \quad x = A_1 e^{+i\omega_0 t} + A_2 e^{-i\omega_0 t} \quad \text{where} \quad A_2 = A_1^*, \quad (* = \text{complex conjugate}) \text{ so}$$

$$A_1 = a - ib \quad \rightarrow \quad A_2 = a + ib \quad \text{(only 2 independent consts.)}$$

Simple harmonic oscillator



- SHM: position and speed of mass
  - Initial conditions give constants needed

$$\underline{Initial\ conditions}: \quad At\ t=0, \quad x=x_0, \quad speed \quad u_0 = \left.\frac{dx(t)}{dt}\right|_{t=0}$$

$$\underline{Substitute\ in\ II}: \quad x = A\cos(\omega_0 t+\phi_0) \quad \rightarrow \quad x_0 = A\cos(\phi_0), \quad u_0 = -A\omega_0\sin(\phi_0)$$

$$x_0^2 = A^2\cos^2(\phi_0), \quad u_0^2 = -A^2\omega_0^2\sin^2(\phi_0) \quad \rightarrow \quad A^2(\cos^2(\phi_0)+\sin^2(\phi_0)) = x_0^2 + (u_0/\omega_0)^2$$

$$A = x_0/\cos(\phi_0) = -u_0/\sin(\phi_0) \quad \rightarrow \quad -u_0/\omega_0 x_0 = \tan(\phi_0)$$

$$So \quad A = \sqrt{x_0^2+(u_0/\omega_0)^2}, \quad \phi_0 = \tan^{-1}(-u_0/\omega_0 x_0)$$

$$\underline{Substitute\ in\ III}: \quad x = A_1 e^{+i\omega_0 t} + A_2 e^{-i\omega_0 t} \quad \rightarrow \quad x_0 = A_1 + A_2, \quad u_0 = (A_1 - A_2)i\omega_0$$

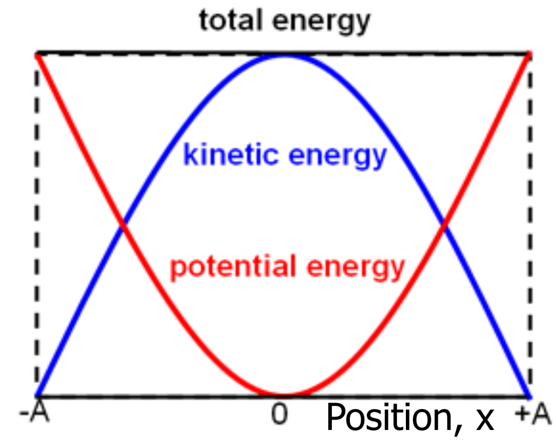
$$So \quad A_1 = \frac{1}{2}(x_0 - i\frac{u_0}{\omega_0}), \quad A_2 = \frac{1}{2}(x_0 + i\frac{u_0}{\omega_0}) \quad \rightarrow \quad x(t) = x_0\cos(\omega_0 t) + \frac{u_0}{\omega_0}\sin(\omega_0 t)$$

$$x_0, \quad u_0 \quad real \quad \rightarrow \quad \mathrm{Im}(x(t)) = 0, \quad and\ \mathrm{Re}(x(t)) = a\cos(\omega_0 t) - b\sin(\omega_0 t)$$

$$Treating\ x, u\ and\ A\ as\ complex, can\ write\ solution\ compactly\ as\ x = Ae^{+i\omega_0 t}$$

$$Then\ speed\ u = \frac{dx}{dt} = i\omega_0 x = i\omega_0 Ae^{+i\omega_0 t}, acceleration\ a = \frac{du}{dt} = -\omega_0^2 x = -\omega_0^2 Ae^{+i\omega_0 t}$$

Simple harmonic oscillator



- Simple harmonic oscillator: energy

$$Energy: E = E_{POT} + E_{KIN}$$

$$with \quad x = A\cos(\omega_0 t + \phi_0), \quad u = -U\sin(\omega_0 t + \phi_0), \quad U = A\omega_0$$

$$E_{POT} = \int_0^x sx\,dx = \frac{1}{2}sx^2 = \frac{1}{2}sA^2\cos^2(\omega_0 t + \phi)$$

$$E_{KIN} = \frac{1}{2}mu^2 = \frac{1}{2}mU^2\sin^2(\omega_0 t + \phi)$$

$$E = \frac{1}{2}sA^2\cos^2(\omega_0 t + \phi) + \frac{1}{2}mU^2\sin^2(\omega_0 t + \phi);$$

$$\omega_0^2 = s/m \quad \rightarrow \quad E = \frac{1}{2}m\omega_0^2 A^2\cos^2(\omega_0 t + \phi) + \frac{1}{2}m(A\omega_0)^2\sin^2(\omega_0 t + \phi)$$

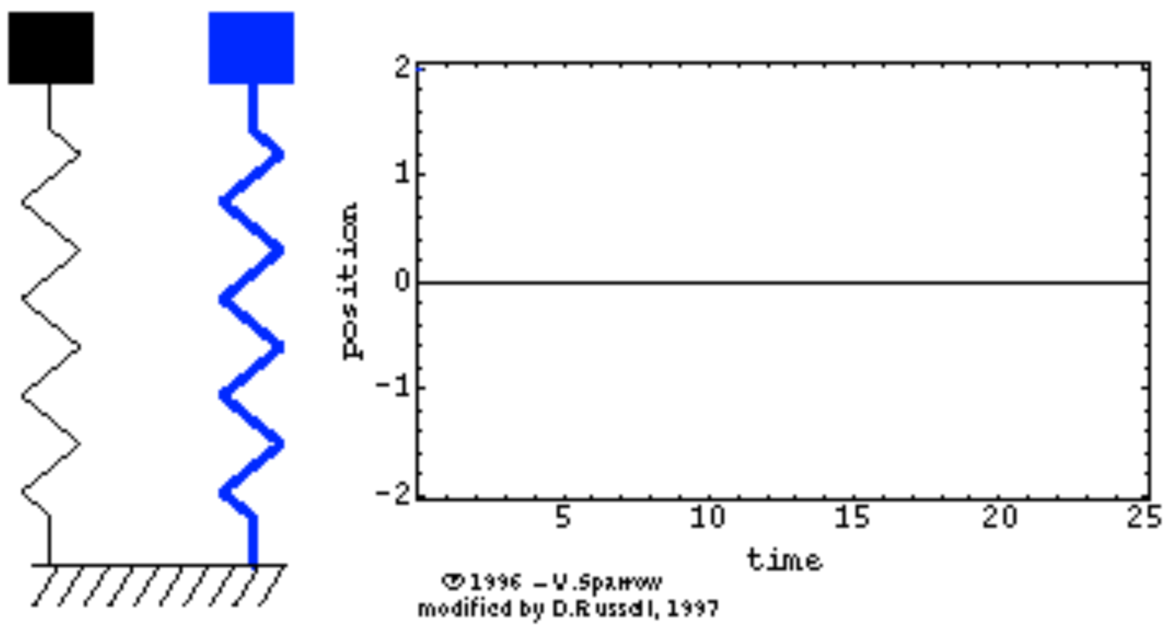
$$E = \frac{1}{2}m\omega_0^2 A^2, \quad but\ also \quad E = \frac{1}{2}sA^2 = \frac{1}{2}mU^2$$

So SHM's total E is constant, and equal to either its max kinetic E (E at $x=x_0$) or max potential E (E at x_{max})

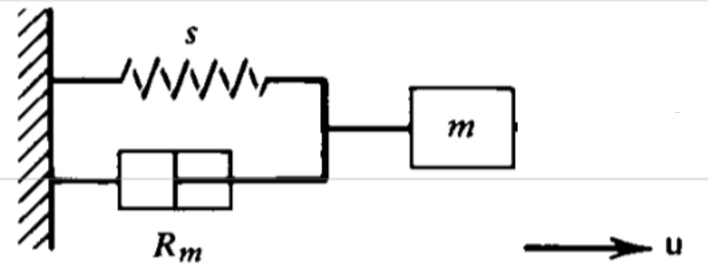
} Undamped, Undriven

Damped harmonic oscillator

- Next, more realistic case: damped harmonic oscillator
  - Damping = friction or resistance → dissipates energy in oscillator
  - Total energy (K + V) decreases with time until oscillator stops
- Compare behavior of free vs damped oscillator:



Damped harmonic oscillator



- Damped oscillator

$$f_{SPRING} = ma \rightarrow \quad -sx = m\frac{d^2x}{dt^2} \quad \rightarrow \quad \frac{d^2x}{dt^2} + \frac{s}{m}x = 0 \quad \text{free oscillator}$$

Damping force (resistance) opposes motion, most commonly proportional to speed:

$$f_r = -R_m\frac{dx}{dt} \rightarrow m\frac{d^2x}{dt^2} + R_m\frac{dx}{dt} + sx = 0 \quad \text{damped oscillator}$$

General complex form of solution: $x = Ae^{\gamma t}$

Put this into eqn: $m\frac{d^2x}{dt^2} + R_m\frac{dx}{dt} + sx = 0 \rightarrow \left(\gamma^2 + \frac{R_m}{m}\gamma + \frac{s}{m}\right)Ae^{\gamma t} = 0$

Must be true for any t, so : $\left(\gamma^2 + \frac{R_m}{m}\gamma + \frac{s}{m}\right) = 0;$ recall natural f : $\omega_0^2 = \frac{s}{m}$

Solution of quadratic in γ : $\gamma = -\beta/2 \pm \sqrt{\beta^2 - 4\omega_0^2}/2;$ with $\beta = R_m/2m$

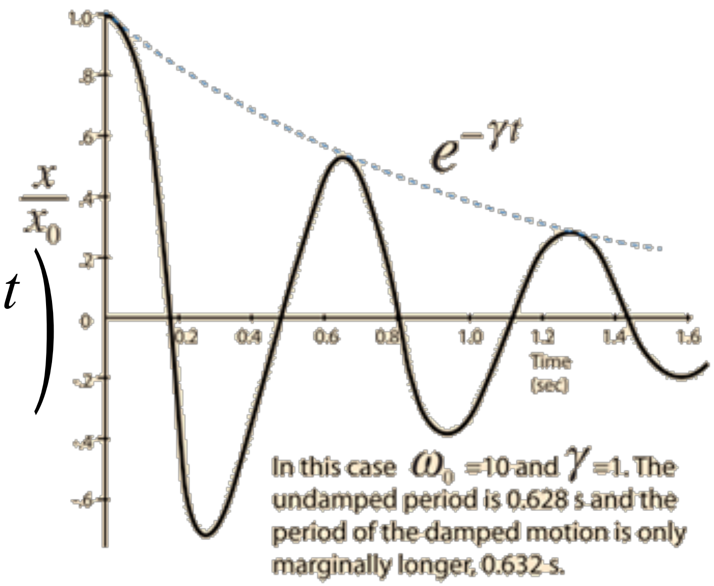
Damped oscillators

- Notice: If $R_m = 0$, we recover undamped case, and;

$$\gamma = \pm\sqrt{-\omega_0^2} = \pm i\omega_0$$

- Also, in most acoustics applications damping is relatively weak, so $\omega_0 > \beta$; define ω_d

$$\omega_d = \sqrt{\omega_0^2 - \beta^2} \quad \rightarrow \quad \gamma = -\beta \pm i\omega_d$$



So solution is $x = e^{-\beta t}\left(A_1 e^{i\omega_0 t} + A_2 e^{-i\omega_0 t}\right)$

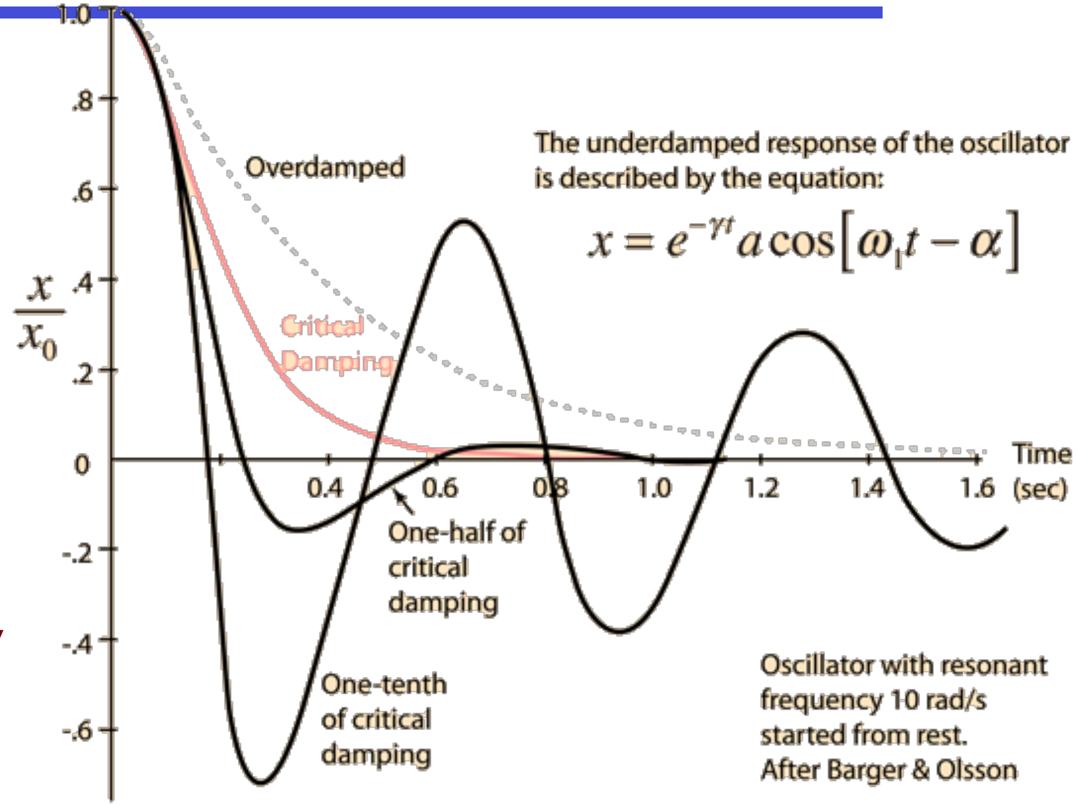
With real part $x = Ae^{-\beta t}\left(\cos(\omega_0 t) + \phi\right)$;

Get the real constants A, ϕ from initial conditions.

- Exponential envelope has decay factor β, so the characteristic time for damping (time to drop by factor 1/e) $\tau_d = 1/\beta = 2m/R_m$

Damped harmonic oscillator

- If damping (R_m) is small, oscillations continue, with slowly diminishing amplitude
- If damping is large, no oscillations, just slow return to equilibrium x
- If damping is "juuust right" (critical damping), returns to equilibrium x=0 smoothly and in shortest possible time
  - Near-critical damping: few oscillations, and a relatively prompt return

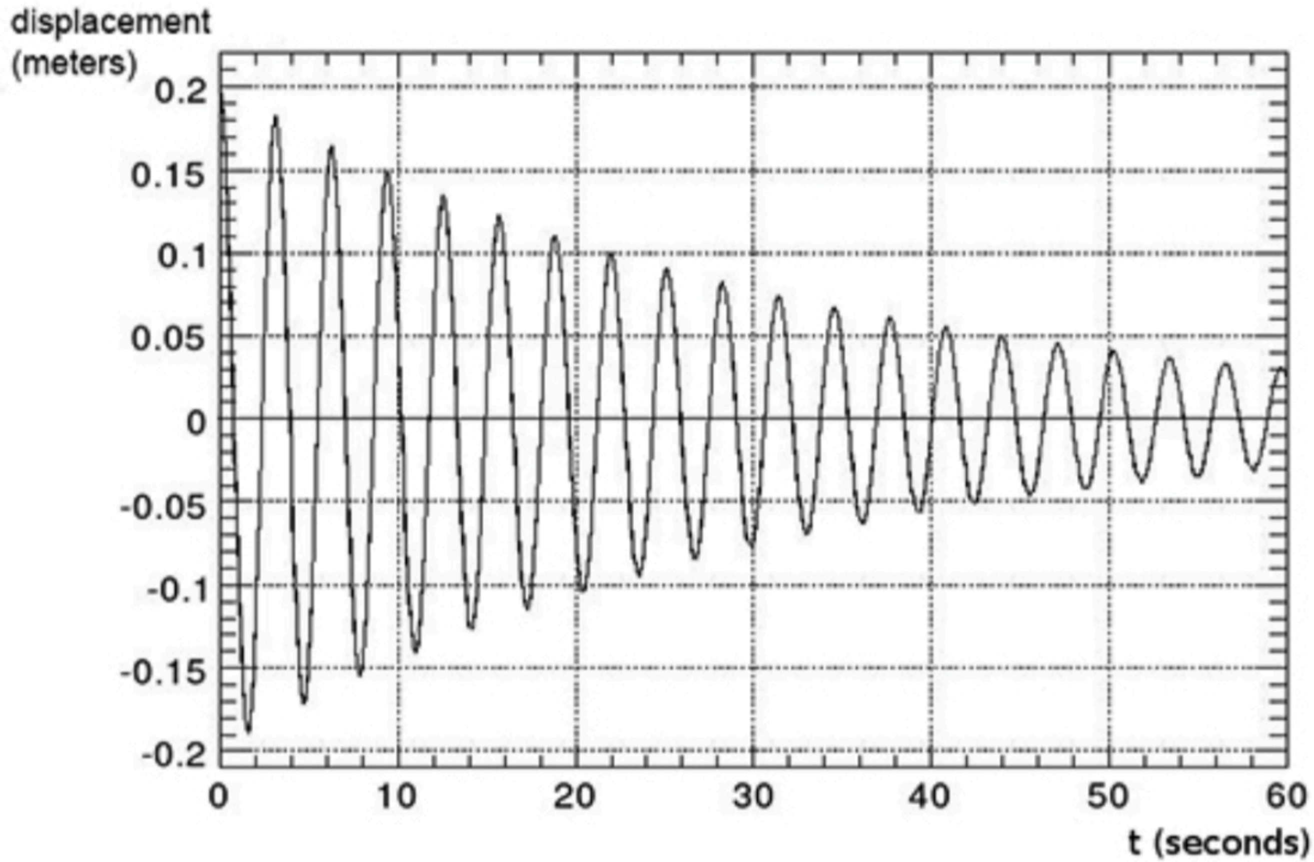


hyperphysics.phy-astr.gsu.edu

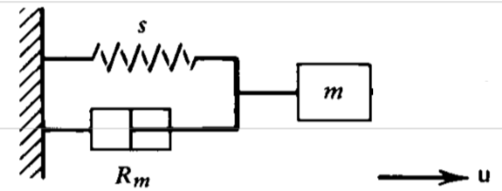
critical damping occurs when $R_m = 2m\omega_0$

Damped harmonic oscillator example

- At t = 0, damped harmonic oscillator is displaced from equilibrium by 0.2 m, where F=4 N, and released. Displacement vs t is shown below:
  - Estimate (a) The mass of the oscillator, (b) The decay factor β, (c) The frequencies ω_d and ω_0



Damped harmonic oscillator example



$$F = -kx \;\rightarrow\; k = 4N/0.2m = 20N/m$$

19 cycles in 60 sec so period $T = 3.15s \;\rightarrow\; T = \frac{1}{f} = \frac{2\pi}{\omega_d} = \frac{2\pi}{\sqrt{\omega_0^2 - \beta^2}}$

$$\omega_0^2 = k/m \;\rightarrow\; T = \frac{2\pi}{\sqrt{(k/m) - \beta^2}} \;\rightarrow\; \omega_d = \frac{2\pi}{T} = 2rad/s$$

$decay: A(t)/A(0) = e^{-\beta t}$; Initial amplitude A(0)=0.2m drops by 1/2 at 20s

$$\rightarrow\; 1/2 = e^{-\beta(20)} \;\rightarrow\; -\ln 2 = -\beta(20s) \rightarrow\; \beta = \ln 2/(20s) = 0.035$$

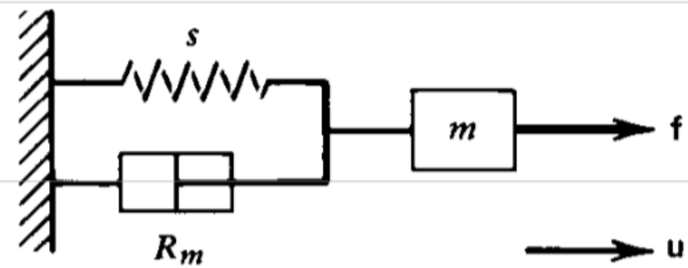
$$\omega_d = \sqrt{(k/m) - \beta^2} \;\rightarrow\; \omega_d^2 = (k/m) - \beta^2 \;\rightarrow\; k/m = \omega_d^2 + \beta^2$$

$$m = k/\left(\omega_d^2 + \beta^2\right) = 20/\left(4\pi^2/T^2 + 0.035^2\right) \sim 5kg$$

$$\omega_0 = \sqrt{k/m} = \sqrt{20/5} = 2rad/s \rightarrow\; f = \omega_0/2\pi = 0.32Hz$$

- Notice that β is small, so $\omega_d \sim \omega_0$

Oscillator with driving force



- Driven oscillator: force applied to m

$$m\frac{d^2x}{dt^2}+R_m\frac{dx}{dt}+sx=f(t)=F\cos(\omega t)$$ driving force applied to oscillator

(Assume F is sinusoidal)

$f = Fe^{i\omega t}$ $\rightarrow$ Again assume general complex form of solution: $x(t)=Ae^{i\omega t}$

Put into $m\frac{d^2x}{dt^2}+R_m\frac{dx}{dt}+sx=f(t) \rightarrow \left(-A\omega^2 m+iA\omega R_m+As\right)e^{i\omega t}=Fe^{i\omega t}$

Solve for A : $A=\frac{F}{i\omega\left(i\omega m+R_m-is/\omega\right)}$ $\rightarrow$ $$x(t)=\frac{Fe^{i\omega t}}{i\omega\left[R_m+i\left(\omega m-s/\omega\right)\right]}$$

Speed : $u(t)=\frac{\partial x}{\partial t}=\frac{Fe^{i\omega t}}{\left[R_m+i\left(\omega m-s/\omega\right)\right]}$

x and u are still treated as complex quantities: define *complex mechanical impedance*

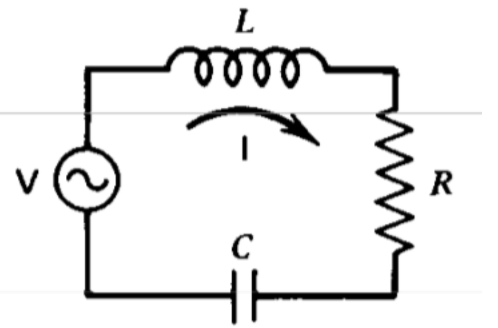
$Z_m=R_m+iX_m$; *mechanical reactance* $X_m=\left(\omega m-s/\omega\right)$

Analogy - Electrical oscillators: RLC

- In electrical circuits, for the driven oscillator:

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{1}{C}q=V(t)$$ driving voltage applied to RLC circuit

Solution for charge $\rightarrow q(t)=\dfrac{V(t)e^{i\omega t}}{i\omega\left[R+i\left(\omega L-1/\omega C\right)\right]}$



Current : $\mathrm{I}(t)=\dfrac{Fe^{i\omega t}}{Z}$ with Z the *complex* electrical *impedance*

$$Z=R+iX=|Z|e^{i\Theta};\ \ \mathrm{Re}[Z]=R;\ \ \mathrm{Im}[Z]=reactance\ \ X=\left(\omega L-1/\omega C\right)$$

$$|Z|=\sqrt{R^2+\left(\omega L-1/\omega C\right)^2};\ \ \Theta=\tan^{-1}\frac{X}{Z};$$ Resonant frequency $\omega_0=\sqrt{\dfrac{1}{LC}}$

  - This is a *series* RLC circuit: same *I* through all components
  - In mechanical oscillator: same *x* and *u* for all parts

Driven damped mechanical oscillator

- Analogy to electrical circuits: mechanical impedance

complex impedance $Z_m = R_m + iX_m = |Z_m| e^{i\Theta}$

$$\mathrm{Re}\left[Z_m\right] = R_m; \qquad \mathrm{Im}\left[Z_m\right] = \text{reactance } X_m = \left(\omega m - s/\omega\right)$$

$$\left|Z_m\right| = \sqrt{R_m^2 + \left(\omega m - s/\omega\right)^2},$$

$$\Theta = \tan^{-1}\frac{X_m}{Z_m}$$

Z_e = ratio of V to I

Z_m = ratio of driving force to speed

"mechanical ohm" has units of force/speed

$$\text{Displacement}: \quad x(t) = \frac{Fe^{i\omega t}}{i\omega\left[R_m + i\left(\omega m - s/\omega\right)\right]}$$

$$\text{Speed}: \quad u(t) = \frac{Fe^{i\omega t}}{\left[R_m + i\left(\omega m - s/\omega\right)\right]} = \frac{f(t)}{Z_m} \quad \rightarrow Z_m = \frac{f(t)}{u(t)}$$

Driven damped mechanical oscillator

- Driven oscillator differential eqn and solutions:

$$m\frac{d^2x}{dt^2}+R_m\frac{dx}{dt}+sx=f(t)=Fe^{i\omega t}$$ ..but actual displacement has to be real!

$\rightarrow$ complex form of solution $x=\dfrac{Fe^{i\omega t}}{i\omega\left[R_m+i\left(\omega m-s/\omega\right)\right]}=\dfrac{f}{i\omega Z_m}$

Actual displacement : $\mathrm{x}(t)=\mathrm{Re}\left[\dfrac{f}{i\omega Z_m}\right]=\dfrac{Fe^{i\omega t}}{i\omega|Z_m|e^{i\Theta}}=\dfrac{-iFe^{i(\omega t-\Theta)}}{\omega|Z_m|}=\dfrac{F\sin(\omega t-\Theta)}{\omega|Z_m|}$

$\left(\text{Recall:}\quad |Z_m|=\sqrt{R_m^2+\left(\omega m-s/\omega\right)^2},\quad \Theta=\tan^{-1}\dfrac{X_m}{Z_m}\right)$

Speed : $u(t)=\mathrm{Re}\left[\dfrac{f(t)}{Z_m}\right]=\dfrac{F\cos(\omega t-\Theta)}{|Z_m|}\rightarrow\dfrac{F}{|Z_m|}=$ Amplitude of $u(t)=$ max speed

  - Notice: find the (complex) impedance and you have solved the differential equation of motion

Transient response

- Transient response of driven, damped oscillator: Full solution includes
  - Transient term that depends on initial conditions, and damping
  - Steady-state term that depends on driving force magnitude, and ω

General complex solution $x = Ae^{-\beta t}e^{i\omega_d t} + \dfrac{Fe^{i\omega t}}{i\omega Z_m}$

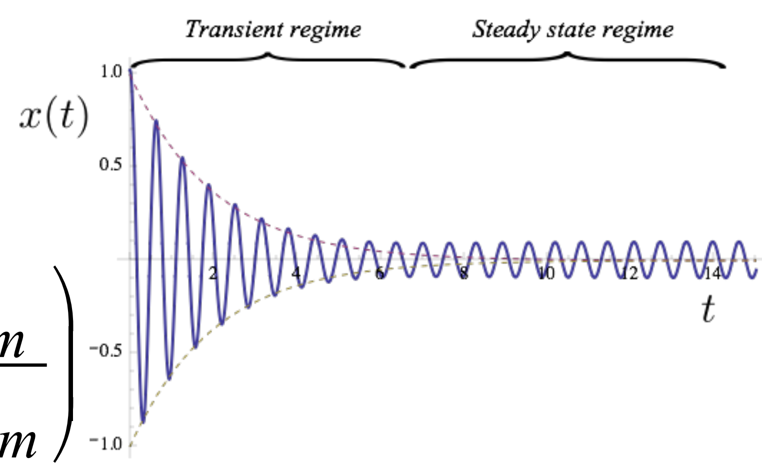
With real part $x = Ae^{-\beta t}\cos(\omega_d t + \phi) + \dfrac{F}{\omega Z_m}\sin(\omega t - \phi)$

We get the real constants $A, \ \phi$ from initial conditions;

$\omega_d = \sqrt{\omega_0^2 - \beta^2}$; $\beta = R_m/2m$

For $x = u = 0$ at $t = 0$, and $\beta << \omega$, we get

$$A = \frac{F}{Z_m^2}\sqrt{\frac{X_m^2}{\omega^2} + \frac{R_m^2}{\omega_d^2}}, \text{ and } \phi = \tan^{-1}\left(\frac{\omega R_m}{\omega_d X_m}\right)$$



Resonance

- Resonance in a driven, damped oscillator:
  - Resonant frequency = f at which reactance *X* goes to 0 *(Z=R)*
    - Minimum of Z → maximum amplitude of speed u

At resonance $\omega_0 m = s/\omega_0 \quad \rightarrow \omega_0^2 = s/m$

$$u_{res} = \frac{F\cos(\omega_0 t)}{R_m} \quad \rightarrow \quad \text{Amplitude } \frac{F}{R_m} = \text{max speed}$$

$$\text{displacement : } \quad x(t) = \frac{F\sin(\omega_0 t)}{\omega_0 R_m}$$

ω_0 is *not* the max displacement frequency;

max displacement occurs for ω such that

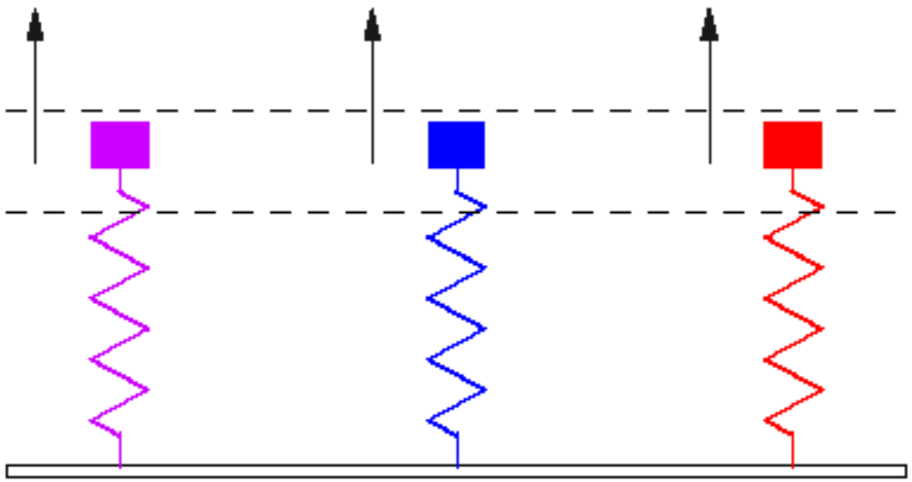
$$\omega\left[R_m^2 + \left(\omega m - s/\omega\right)^2\right] = \text{ minimum} \quad \rightarrow \quad \omega_{\max - x} = \sqrt{\omega_0^2 - 2\beta^2}$$

(...homework exercise)

Resonance

- Resonance in a driven oscillator:
  - Identical damped oscillators, all with natural frequency $f_0=1$.
  - Harmonic driving force $F=F_0\cos(2\pi f t)$ is applied
  - The driving frequencies of the applied forces are (matching colors)

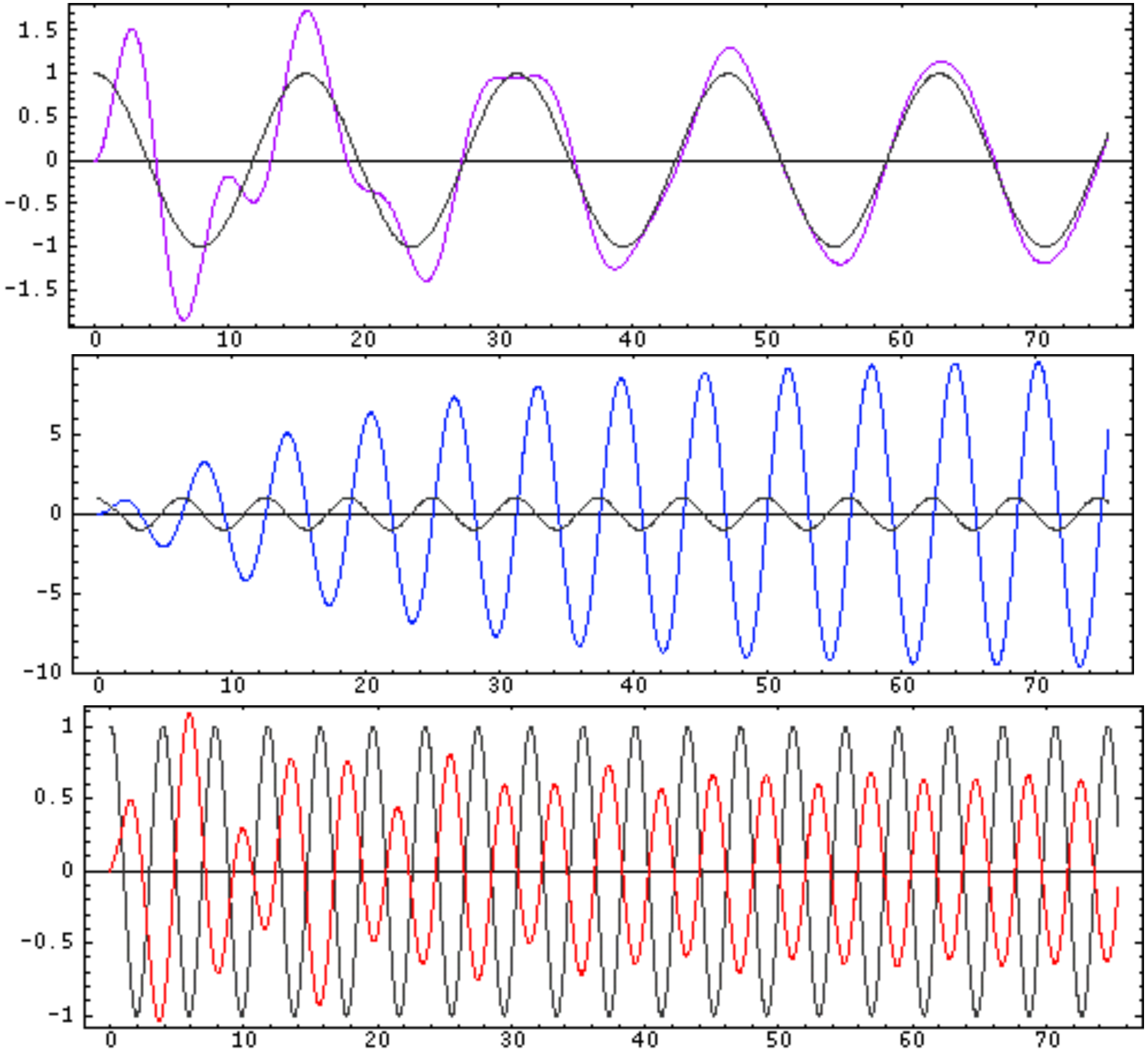
$f1=0.4$, $f2=1.01$, $f3=1.6$

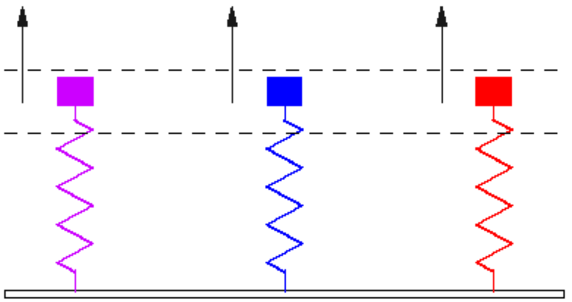


www.acs.psu.edu/drussell/

Resonance

– Time plots for oscillators (matching colors) f1=0.4, f2=1.01, f3=1.6



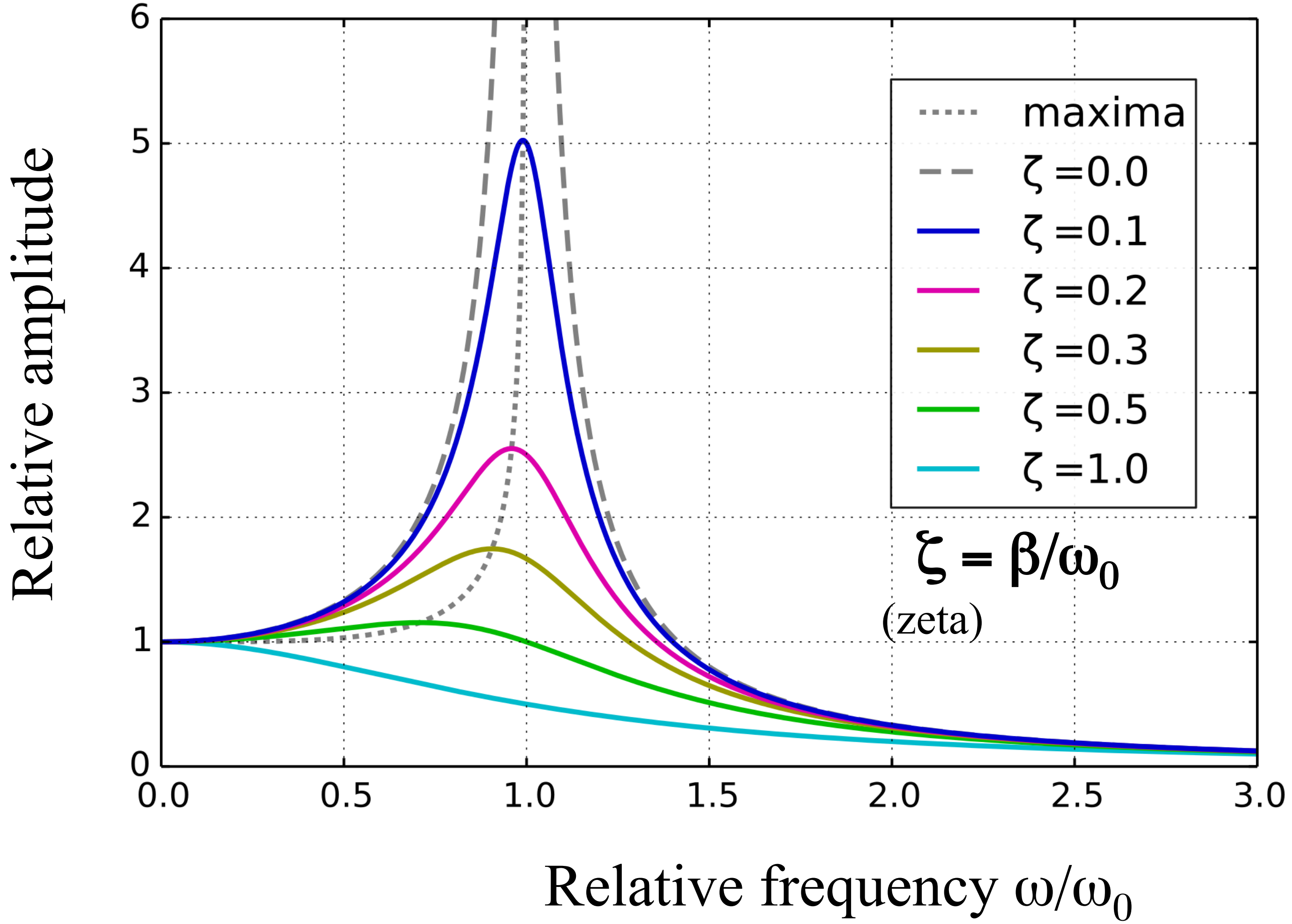


t axis on these plots seems to be 1 sec = ~6 units

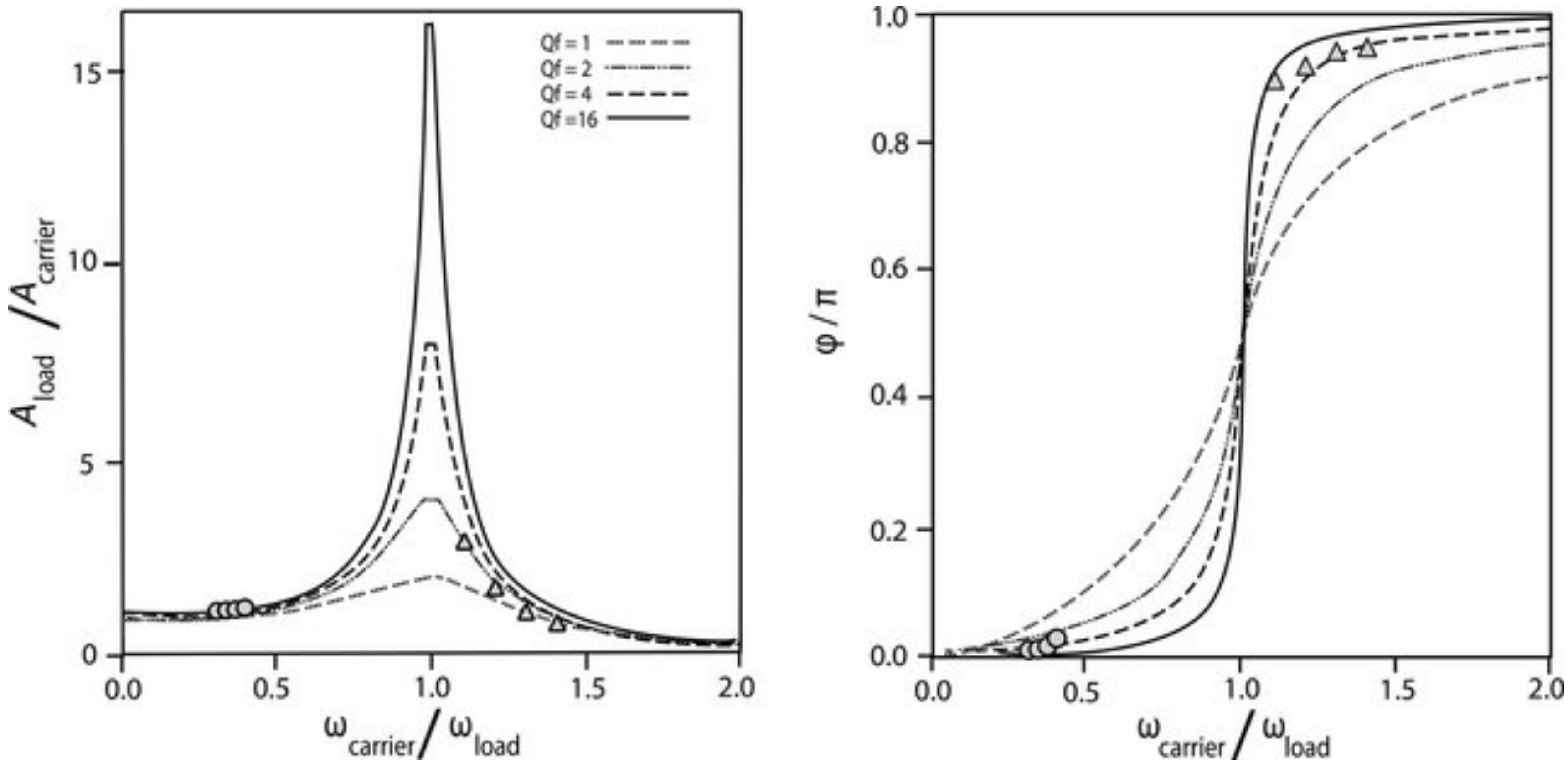
Displacement amplitude vs damping β and ω/ω_0

- Resonance in a driven oscillator: frequency for max displacement

$$\omega_{\max-x} = \sqrt{\omega_0^2 - 2\beta^2}, \qquad \beta = R_m / 2m$$



Phase shift vs (ω / ω_0)



- Resonance amplitude and phase shift in a driven oscillator

Watch https://www.youtube.com/watch?v=aZNnwQ8HJHU

Resonant behavior and Q

- Sharpness of resonance peak is described by the 'quality factor' Q

$$Q = \frac{\omega_0}{\omega_U - \omega_L}, \quad \omega_U,\ \omega_L \text{ are points above and below resonance}$$

where average power drops to 1/2 the resonance peak value.

This happens when $Z_m^2 = 2R_m^2 \rightarrow X_m = \omega m - s/\omega = \pm R_m$

$$\omega_U m - s/\omega_U = R_m, \quad \text{and} \quad \omega_L m - s/\omega_L = -R_m \rightarrow \omega_U - \omega_L = R_m/m$$

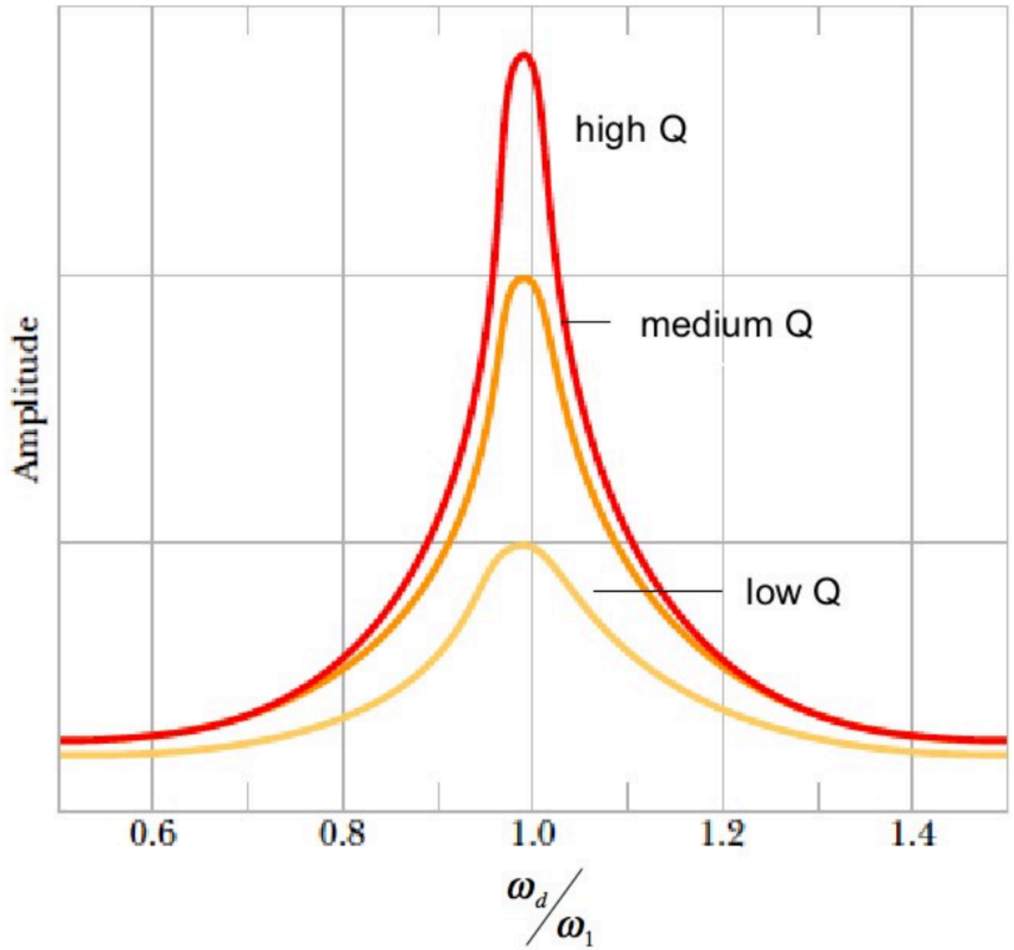
$$\text{So} \quad Q = \frac{\omega_0 m}{R_m} = \frac{\sqrt{mk}}{R_m} = \omega_0(\tau/2) = (2\pi/T)(\tau/2) = \pi\left(\frac{\tau}{T}\right)$$

So

- Q is directly proportional to decay time of the undriven oscillator **τ**
- Q is related to (energy stored)/(energy lost) ~ *X/R*

Resonant behavior and Q

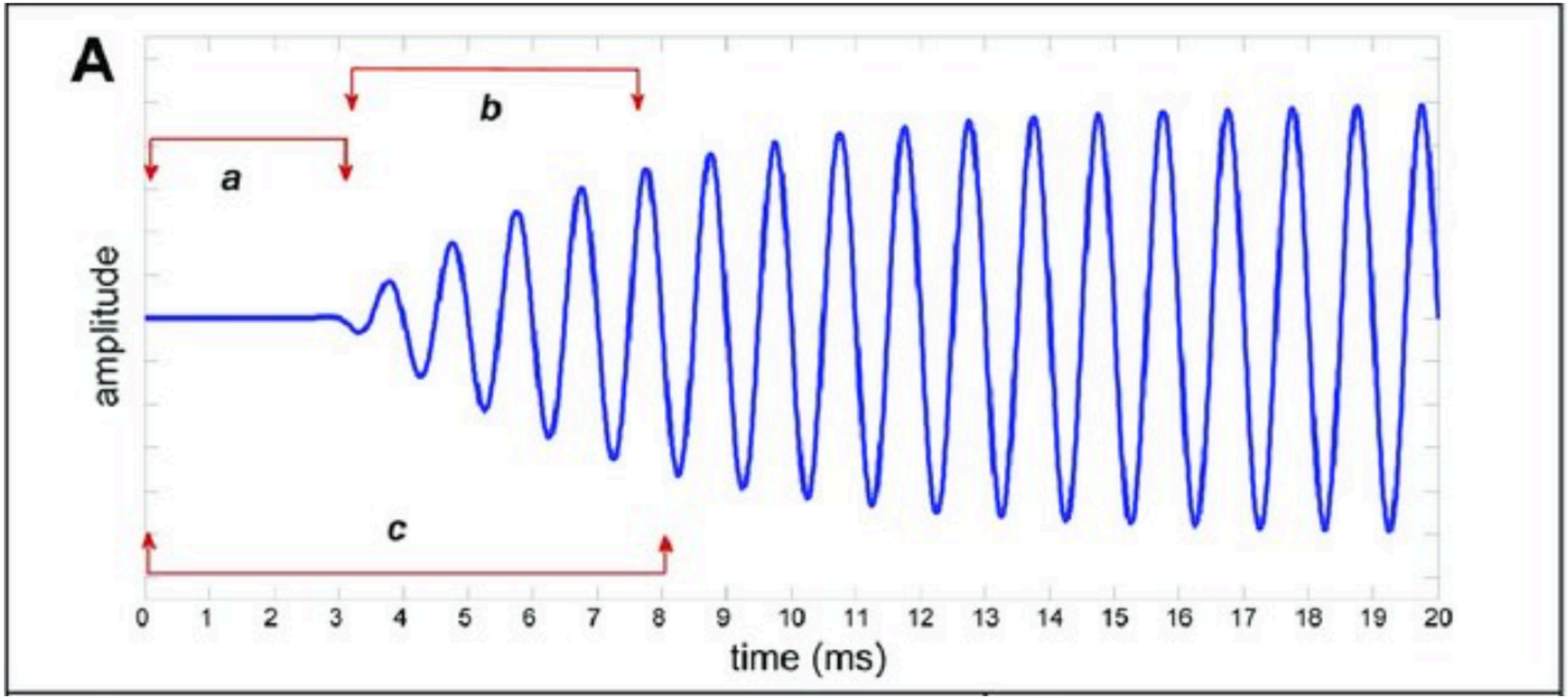
- Q is dimensionless (ratio of times or energies)
- Low resistance (dissipation) → High Q



Displacement amplitude vs time for resonance: human ear

- Example of resonance in a driven oscillator: Amplitude vs t, for natural frequency 1 kHz and Q = 12.7 (values reflecting those of the cochlea). a = delay before start, b = resonance build-up time

The oscillator takes Q/π cycles (~4 cycles) to reach the half-power amplitude of 0.707 and Q cycles to reach the 96% criterion. *- A. Bell, PLoS One 7(11):e47918, 2012*



Stiffness, Resistance, Mass dominance

- Often want some response to be 'flat' over a range of frequencies
  - Stiffness-controlled system

    $s/\omega >> \omega$ m and $R_m \rightarrow x(t) \sim (F/s)\cos(\omega t)$
    - Amplitude of *displacement* ~ independent of ω
  - Resistance-controlled system

    $R_m >> X_m$ and $R_m \rightarrow x \sim (F/\omega R_m)\sin(\omega t),\ u \sim (F/R_m)\cos(\omega t)$
    - Amplitude of *speed* ~ independent of ω
  - Mass-controlled system

    ω m >> s/ω and $R_m \rightarrow x \sim (F/\omega^2 R_m)\cos(\omega t),\ u \sim (F/\omega R_m)\sin(\omega t)$
    - Neither amplitude of *u or x* is independent of ω (but amplitude of *acceleration* is)
- Driven oscillators are
  - stiffness-controlled for $\omega << \omega_0$
  - Resistance-controlled for $\omega \sim \omega_0$
  - mass-controlled for $\omega >> \omega_0$

Discussion of resonance wouldn't be complete…

- ...without the Tacoma Narrows Bridge 1940

https://www.youtube.com/watch?v=XggxeuFDaDU

Reminder

SPECIAL: Next Tuesday, class will start at 7:30 pm, not 7 pm.