PHYS 536

R. J. Wilkes

Session 8

Absorption losses

Pulsating spheres and simple sources

Dipoles and pistons

Waves in pipes

1/26/2023

Course syllabus and schedule – updated

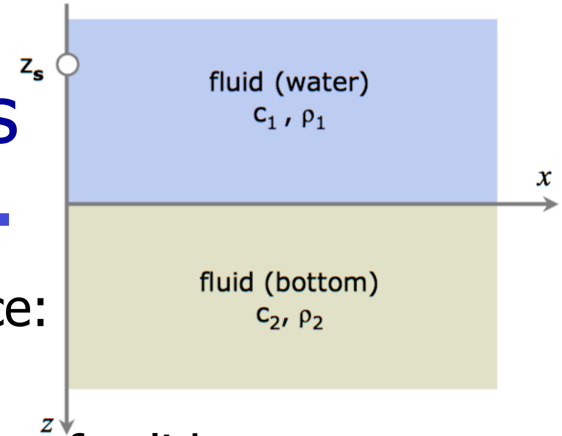
See : http://courses.washington.edu/phys536/syllabus.htm

6	19-Jan	Thu	K. Ch. 5, 6	H: Ch. 1	… of sound; Harmonic plane waves, intensity, impedance. UPDATED BELOW HERE:
7	24-Jan	Tue	K. Ch. 5	H: Ch. 2	Spherical waves; Eikonal equation and ray tracing; transmission and reflection at interfaces;
8	26-Jan	Thu	K: Ch. 7	H: Ch. 7	Absorption losses; Pulsating spheres and simple sources and dipoles
9	31-Jan	Tue	K. Ch. 8-10	H: Chs. 13	Near field, far field; Radiation impedance; waves in pipes; rectangular cavities; Helmholtz resonators;
10	2-Feb	Thu	K. Ch 12		Resonant bubbles; Acoustic impedance; physical acoustic filters; Decibels and sound level measurements
11	7-Feb	Tue	K. Ch 12	H: Ch. 28	Environmental acoustics and noise criteria; industrial and community noise regulations; noise mitigation; frequency, pitch and musical tones
12	9-Feb	Thu		H: Chs. 18-19	Interference effects; Musical acoustics; Musical instruments: strings. REPORT 1 PAPER DUE by 7 PM; REPORT 2 PROPOSED TOPIC DUE
13	14-Feb	Tue	K. Ch. 11	H: Ch. 16	Musical instruments: winds; The ear, and human hearing;
14	16-Feb	Thu	K. Ch. 11	H: Chs. 21-22	Human hearing: the inner ear; pitch perception; acoustics of speech
15	21-Feb	Tue	K. Chs. 13-14	H: Ch. 27; Ch. 6	Room acoustics; Transducers for use in air and water: Microphones and loudspeakers; hydrophones and pingers
16	23-Feb	Thu	K. Ch 15		Underwater acoustics; sound absorption underwater, the sonar equation; applications: acoustical positioning, seafloor imaging, sub-bottom profiling; Doppler effect
17	28-Feb	Tue			Course wrapup. Student report 2 presentations
18	2-Mar	Thu			Student report 2 presentations
19	7-Mar	Tue			Student report 2 presentations
20	9-Mar	Thu			Student report 2 presentations. TAKE-HOME FINAL EXAM ISSUED
--	17-Mar	Fri			FINAL EXAM ANSWERS DUE by 5 PM

Tonight

Class is over after you turn in your take-home exam. No in-person final exam during finals week.

Reflections at solid(ish) surfaces



From last time

- For reflection on a (hard) solid surface, with normal incidence:
  - Ignore transmission: consider only reflected wave
  - Describe behavior in terms of *normal acoustic impedance* of solid

$$z_n = \frac{p}{\vec{u}\cdot\hat{n}}\ , \quad \hat{n} = \text{ unit vector normal to surface}$$

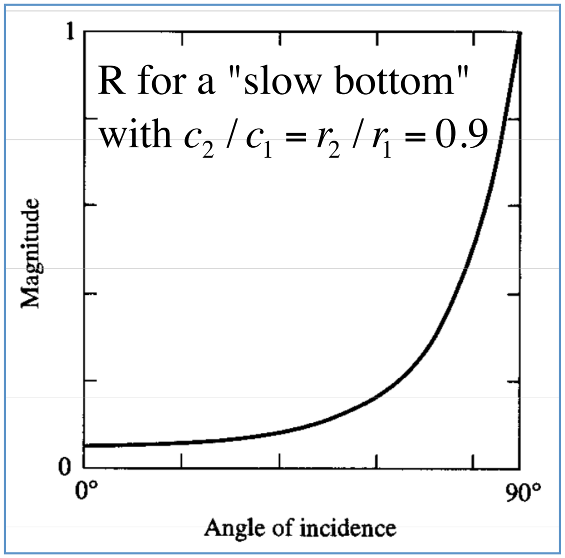
For normal-incidence wave, $\vec{u}\cdot\hat{n} = u_x$ the particle speed

$$z_n = r_n + i\,x_n$$

continuity of p and u at $x=0$: $p_i + p_r = p_t, \quad u_i + u_r = u_t$

$$\rightarrow z_n = \frac{p_i + p_r}{u_i + u_r} \rightarrow z_n = r_1\frac{1+R}{1-R} \rightarrow R = \frac{(r_n - r_1) + i\,x_n}{(r_n + r_1) + i\,x_n}$$

For $x_n = 0$, this is same as for normal incidence on a fluid-fluid interface.



  - For a semi-solid surface (eg seabottom), transmitted power might be significant, in both transverse and shear waves – sound speed must include shear modulus:

$$c = \sqrt{B/\rho_0} \rightarrow \sqrt{(B + (4/3)G)/\rho_0}\,, \quad G = \text{shear modulus}$$

Absorption losses

- Waves in a medium with viscosity lose energy:
  - Viscosity means time is required to reach equilibrium after δp
    - Energy of wave motion → thermal energy
  - Describe behavior in terms of linearized equation of motion

ignoring losses, we have $p = \rho_0 c^2 s$: pressure and compression are in phase

modify equation of state $p = \rho_0 c^2 \left(1 + \tau \frac{\partial}{\partial t}\right) s$ where τ is relaxation time

Assume fluid is at rest until $t = 0$ when sudden increase of pressure P_0 is applied

solution that satisfies this condition is $= s = \frac{P_0}{\rho_0 c^2}\left(1 - \exp(-t/\tau\right) s, \quad t > 0$

$$\nabla \cdot \vec{u} = \frac{1}{\rho_0}\frac{\partial p}{\partial t} = -\frac{\partial s}{\partial t}, \quad \text{and} \quad \rho_0 \frac{\partial \vec{u}}{\partial t} = -\nabla p \rightarrow \quad \left(1 + \tau_S \frac{\partial}{\partial t}\right)\nabla^2 p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}, \quad \text{(Lossy wave eqn)}$$

$\tau_S = \left(\left(4/3\right)\eta + \eta_B\right)\rho_0 \left(\frac{\partial \vec{u}}{\partial t}\right),$ η and $\eta_B =$ coefficients of shear and bulk viscosity

$c^2 = \left(\frac{\partial \mathcal{P}}{\partial \rho}\right)_{ADIAB}$; not the phase speed c_P, due to τ_S : $\mathcal{P}$ = hydrostatic pressure

Absorption losses

– in terms of linearized equation of motion

for monofrequency case, $p \sim \exp(i\omega t)$, wave eqn $\rightarrow$ "Lossy Helmholtz eqn"

$$\nabla^2 p + \mathrm{k}^2 p = 0, \quad \text{with } \mathrm{k} = k - i\alpha_S = (\omega / c)\left(1 + i\omega\tau_S\right)^{1/2}$$

$$\rightarrow \quad \alpha_S = (\omega / c\sqrt{2})\left[\frac{\sqrt{1+\left(\omega\tau_S\right)^2} - 1}{1+\left(\omega\tau_S\right)^2}\right]^{-1/2}, \quad c_P = (\omega / k) = c\sqrt{2}\left[\frac{1+\left(\omega\tau_S\right)^2}{\sqrt{1+\left(\omega\tau_S\right)^2}}\right]^{+1/2}$$

$$\vec{p} = P_0 \exp(i(\omega t - \vec{k}x)) \rightarrow \; p = P_0 \exp(i(\omega t - kx))\exp(-\alpha x)$$

traveling wave has phase speed $c_P = (\omega / k)$ not $= c$ but a $f\,(freq)$

acoustic impedance is $\vec{z} = (\vec{p} / \vec{u}) = \rho_0 c_P \dfrac{1}{\left(1 - i\alpha / k\right)}$

Lossy propagation

For wave in +x direction

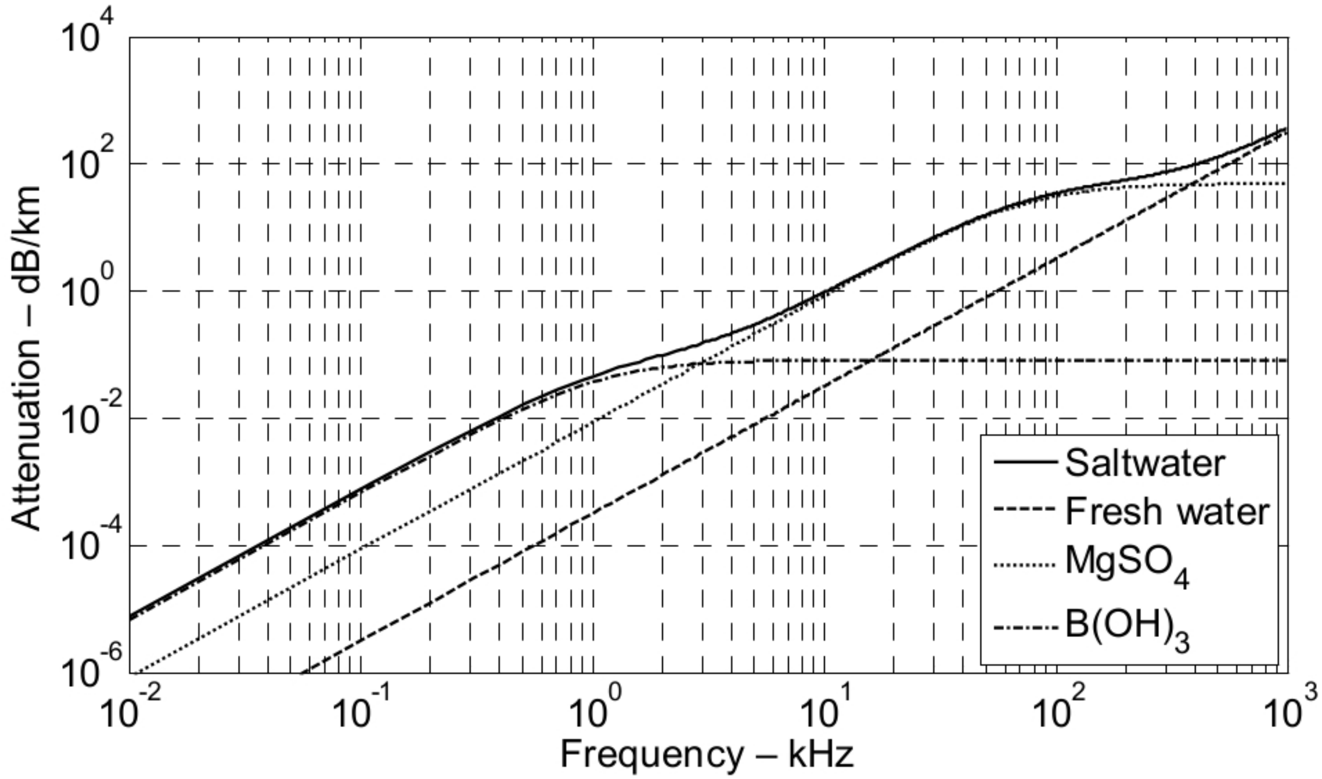
$$p_+ = P_0 \exp\left[i(\omega t - \mathrm{k}\,x)\right] = P_0 \exp(-\alpha_S x)\exp\left[i(\omega t - kx)\right]$$

Wave amplitude decays in x direction $\rightarrow$ α_S = *spatial absorption coefficient*.

The phase speed c_P is frequency-dependent $\rightarrow$ propagation is *dispersive*.

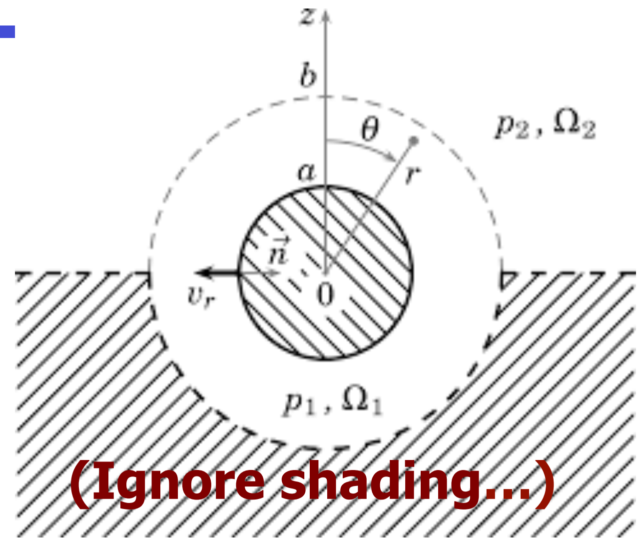
Typical values of $\tau_S \sim 10^{-10}s$ for gases, $10^{-12}s$ for liquids,

so $\omega\tau_S << 1$ except at ultrasound frequencies.



From *http://dx.doi.org/10.5772/55935*

Sound sources: pulsating sphere



- Sphere pulsating in radial direction
  - Model for "simple" ~point sources
    - Assume |δr| << radius a
  - Surface speed *u= U_0 exp(iωt)*
  - Boundary conditions:
    - No reflection at ∞ → diverging only

      p(r,t)= (A/r) exp[i (ωt-kr)]
  - Acoustic impedance seen at r=a:

$$\text{Recall,}\quad \Phi_+(r,t) = -\frac{\vec{p}}{i\omega\rho_0},\quad \vec{u} = \nabla\Phi = \left(1 - \frac{i}{k|r|}\right)\frac{\vec{p}}{i\omega\rho_0}$$

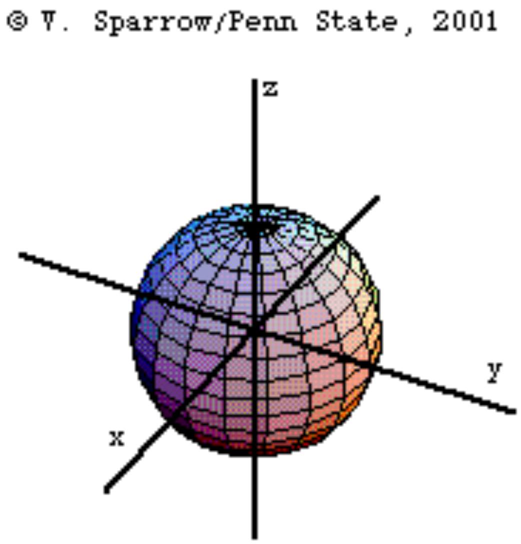
Observable quantities are real parts:

$$p(r,t) = \mathrm{Re}\left(\frac{A}{r}\exp(i\omega t - kr)\right), u(r,t) = \mathrm{Re}\left(=\left(1 - \frac{i}{k|r|}\right)\frac{\vec{p}}{i\omega\rho_0}\right)$$

So u is not in phase with p, and z is not just ρc

$$z = \frac{p}{u} = \rho_0 c\frac{kr}{\sqrt{1+(kr)^2}}\exp(i\theta),\ \text{where}\ \cot\theta = kr \rightarrow z(a) = \rho_0 c\cos\theta\exp(i\theta_a)$$

Sound sources: pulsating sphere



- Sphere pulsating in radial direction
  - Pressure at source surface r=a:

    $p(r,t) = (A/r)\exp[i(\omega t - kr)]$

    $p(a,t) = \rho_0 c U_0 \cos(\theta_a)\exp[i(\omega t + \theta_a)]$

    So $A = \rho_0 c\, U_0\, (a/r)\cos(\theta_a)$, and for $r>a$,

    $p(r,t) = \rho_0 c\, U_0\, (a/r)\cos(\theta_a)\exp[i(\omega t - k(r-a) + \theta_a)]$
  - If $a \ll \lambda$, so $ka \ll 1$

$z(a) = \rho_0 c \cos\theta \exp(i\theta)$, where $\cot\theta = ka$

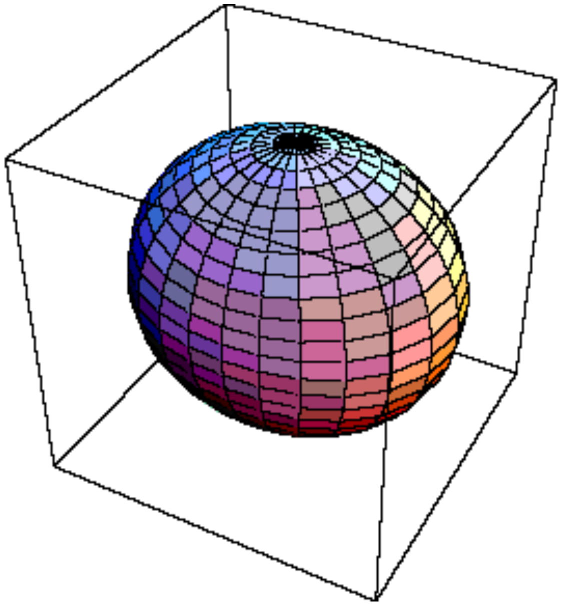
z has reactive part dominant (energy stored as fluid layers pulsate)

so $z(a) \approx \rho_0 c\, ka(i + ka) \rightarrow$ for long λs, pressure at $r > a$ is

$$p(r,t) \approx i\ \rho_0 c U_0 \left(\frac{a}{r}\right) ka\ \exp[i\ (wt + kr)] \rightarrow \text{ nearly } \frac{\pi}{2} \text{ out of phase with } u$$

$$\text{and intensity is } I = \left(\frac{1}{2}\right)\rho_0 c U_0^2 \left(\frac{a}{r}\right)^2 (ka)^2 \rightarrow I \text{ is proportional to } \omega^2 \text{ and } a^4$$

Source strength



Sphere pulsating in (2,2) mode

- Single-frequency source might be non-spherical or have varying values of U and its phase, at different places on surface
  - Still assuming |δr| << radius a
    - "Simple source"
  - We want a general way to characterize source strength

Instantaneous velocity of a given point on the surface is

$$\vec{u}(r=a;\, t,\varphi) = \vec{U}\exp\left[i(\omega t-\varphi)\right]$$

Source displaces fluid volume at the rate

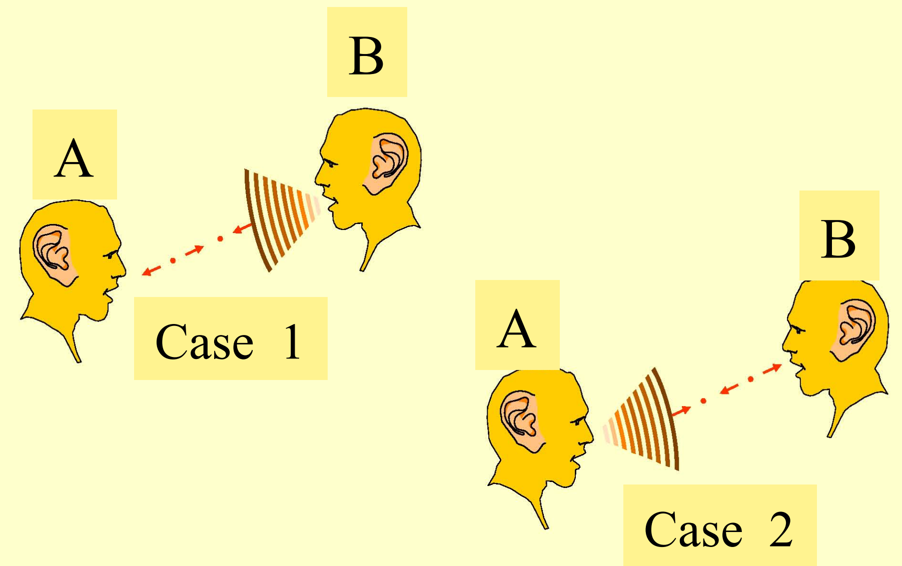
$$Q\exp(i\omega t+\varphi) = \int_S \vec{u}\cdot\hat{n}\,dS \quad \text{complex source strength}$$

(For a uniformly pulsating sphere, Q is real: $Q = 4\pi a^2 U_0$)

Acoustic reciprocity

- Useful analysis tool is acoustic reciprocity theorem:
  - If medium is not moving in bulk, source and receiver positions can be swapped with no change in results
  - Example: Two sources A, B
  - Case 1: A=source, B=receiver
  - Case 2: B=source, A=receiver

Solution for p(r=A,t) in case 1 will be same as p(r=B,t) in case 2



$$p_A(\mathbf{x}_B, t) = p_B(\mathbf{x}_A, t)$$

http://www.falstad.com/ripple/

Color sch 6 Ex: slow medium, anti-refl, refr, int refr,
Reduce speed, damping, incr resolution
Move boxes up
Ex: reciprocity

Acoustic reciprocity

- Acoustic reciprocity theorem in more technical terms:
  - As before, two transducers: 1 and 2 - one is source, other receiving:

Recall - $\vec{u} = \nabla\Phi$ where Φ is a scalar *velocity potential*

for spherical waves, $\Phi \propto p$, and wave eqn $\rightarrow \nabla^2\Phi = -k^2\Phi$

so for 2 sources (only one active), we can apply Green's Theorem (see Kinsler 8.3-4)

to get $\int_S \left(\Phi_1\nabla\Phi_2 - \Phi_2\nabla\Phi_1\right)\cdot\hat{n}\,dS = \int_V \left(\Phi_1\nabla^2\Phi_2 - \Phi_2\nabla^2\Phi_1\right)dV$

$\rightarrow \int_{S=SOURCE} \left(p_1\vec{u}_2 - p_2\vec{u}_1\right)\cdot\hat{n}\,dS = 0,$ where S encloses whichever source is active.

$\rightarrow \int_S \left(p_1\vec{u}_2\right)\cdot\hat{n}\,dS = \int_S \left(p_2\vec{u}_1\right)\cdot\hat{n}\,dS$ where S is distant and encloses both 1 and 2.

Source $Qe^{i\omega t} = \int_S \vec{u}\cdot\hat{n}\,dS$, and $p = P(r)e^{i\omega t} \rightarrow \dfrac{Q_1}{P_1(r)} = \dfrac{Q_2}{P_2(r)}$ for "simple sources"

So for large r, pressure field of an irregular simple source *is identical to* pressure field of a pulsating sphere with the same Q

pulsating sphere has $p(r,t) \approx i\ \rho_0 c U_0 \left(\dfrac{a}{r}\right) kae^{i(\omega t - kr)}$,

(where $k = 2\pi/\lambda$, and $a << \lambda$) and $Q = 4\pi a^2 U_0 \rightarrow \dfrac{Q}{P} = \dfrac{2\lambda r}{\rho_0 c}$ Free-field reciprocity factor

Application: reciprocity calibration of microphones

- Primary standard for calibration of measurement microphones
  - For scientific/engineering measurements, precise sensitivity must be known (in volts per Pa)
  - Sensitivity varies with f → calibrate at several frequencies
  - Can use 3 uncalibrated microphones i, j, k (as long as they are reciprocal)

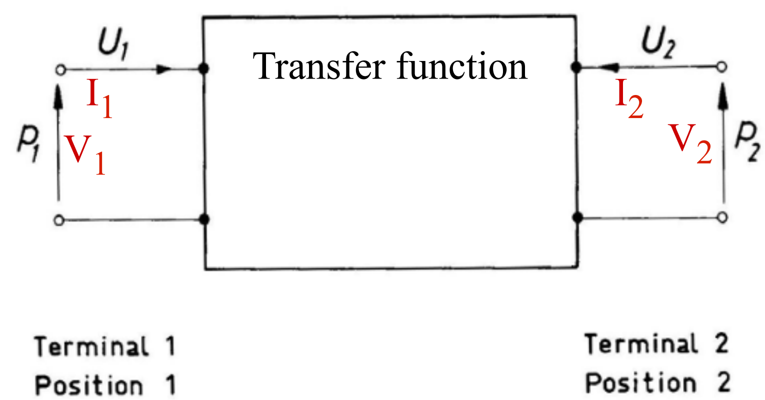
    → sensitivity in V/Pa as a receiver = sensitivity in m³/s/A as transmitter
  - Set up mic i facing mic j, drive i aind measure signal at j
    - Find electrical transfer impedance $Z_{ij} = V_j \,/\, I_i$

      I_i = driving current in mic i

      V_j = output of mic j
    - Sensitivity of mics is related by $Z_{ij} = M_i \; Z_{AC} \; M_j$

      Z_{AC} = acoustical transfer impedance
    - For free-field / far field coupling

$$Z_{\text{ac, free}} = \frac{\rho_0 \omega}{4\pi r} e^{-\frac{m}{2} r} e^{-j(kr - \frac{\pi}{2})}$$



  - Repeat with mics j → k, and k → i
  - Solve 3 eqns for 3 unkn

$$M_i = \sqrt{\frac{1}{Z_{\text{ac}}} \frac{Z_{ij} Z_{ik}}{Z_{jk}}}$$

Application: reciprocity for noise abatement analysis

- For auto/truck tire noise:
  - Place omni transducer inside car and use as source at various f's
  - Use reciprocal mic to measure sound field at points on tire and acoustic baffling in car body
    - → Identify locations that contribute most, or are blocked, to improve blocking
- For auto/truck engine noise:
  - Same procedure, now measure sound field at various points on engine surface and acoustic baffling in car body
    - → Identify locations that contribute most, or are blocked, to improve blocking
- For railway wheel noise:
  - To evaluate and improve design of trackside baffling walls, need to model wheels and railbed as acoustic sources and include diffraction behavior of the barrier.
    - Building a model is difficult: complex geometry of undercarriage; scattering and absorption properties of the ballast and sleepers as well as barrier.
  - Place omni transducer at railside receiver positions and use as source
  - Use reciprocal mic to measure sound field at various points on wheels and undercarriage
  - Find barrier insertion loss via transfer functions measured reciprocally in the presence and absence of the barrier

Application: reciprocity for noise abatement analysis

From: *Some Applications of the Reciprocity Principle in Experimental Vibroacoustics*, F. J. Fahy, Institute of Sound and Vibration Research, U. of Southampton *Acoustical Physics*, Vol. 49, 2003, p. 217.

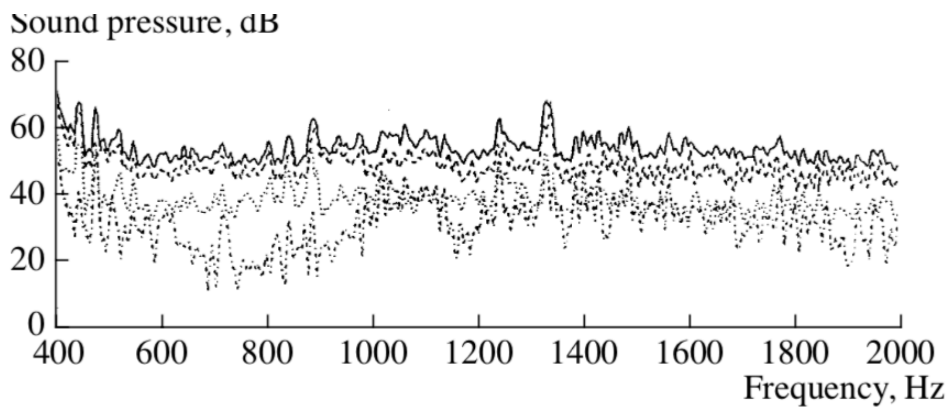


Fig. 5. Estimated sound pressure contributions from various components: (solid line) complete engine, (dashed line) engine block, (dotted line) valve cover, (dash–dotted line) gear box.

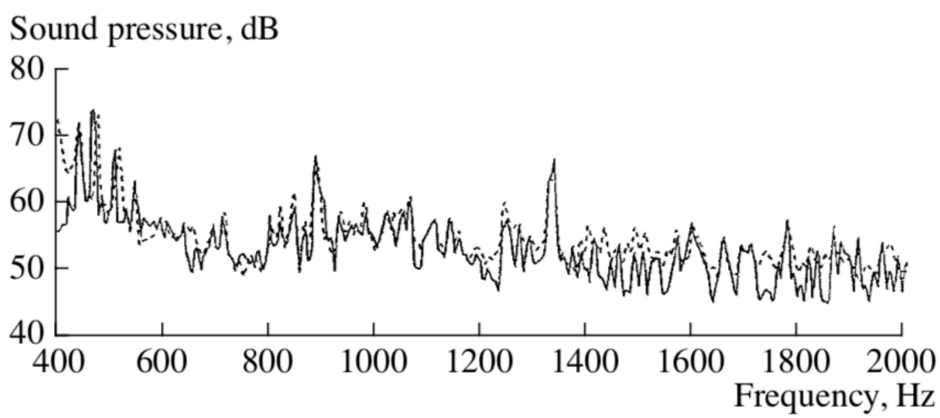


Fig. 6. Comparison of the measured sound pressure level spectrum with that estimated from reciprocity and near-field intensity measurements.

Simple source with hard baffle behind it

- Common case: simple-source transducer mounted on a wall
  - Simple-source = single-*f* source with closed surface, with arbitrary vibration pattern, but all dimensions << λ
  - Hard baffle means no penetration behind source, power radiated backward is reflected

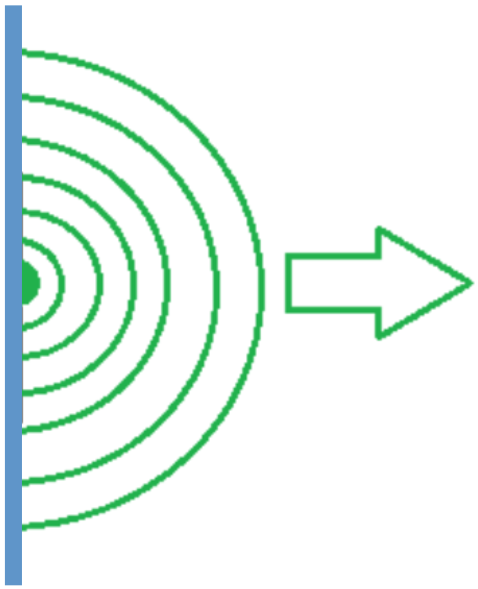
    → 2X pressure and 4X intensity in forward hemisphere

$$\text{so } p(r,t) = 2\left(i\ \rho_0 c U_0 \left(\frac{Qk}{4\pi r} \right) ka \exp[i(wt - kr)] \right) = i\ \rho_0 c U_0 \left(\frac{Qk}{2\pi r} \right) ka \exp[i(wt - kr)]$$

$$\text{and } I(r) = 4\left(\frac{1}{8}\ \rho_0 c \left(\frac{Q}{\lambda r} \right)^2 \right) = \frac{1}{2}\ \rho_0 c \left(\frac{Q}{\lambda r} \right)^2$$

$$\text{Power radiated} = \int_{FWD\ HEMISPHERE} I(r)\,dV = \rho_0 c \left(\frac{Q}{\lambda} \right)^2$$

twice the power compared to isolated simple source (radiating into both hemispheres)



Dipole sources

- Can use multiple simple sources to model more complicated sources: *$p(r,t) = p_1(r,t) + p_2(r,t) + \ldots$*
- Common example: dipole source (acoustic doublet)
  - Two simple sources of same *f* and equal *Q* separated by distance *d*, **but** 180 deg out of phase; then

$p_1(r, \theta, t) = [A/(r_1+\Delta r_1)] \exp[i (\omega t-kr_1 + k\Delta r_1)]$ where Δr_1 = path difference from observation point r to source 1, relative to midpoint distance r, and

$p_2(r, \theta, t) = [A/(r_2+\Delta r_2)] \exp[i (\omega t-kr_2 - k\Delta r_2)]$, so (including 180° phase diff)

$$p(r,t) = \left(\frac{A}{r}\right)\left(\frac{\exp(-ik\Delta r_1)}{1+(\Delta r_1/r)} - \frac{\exp(+ik\Delta r_2)}{1-(\Delta r_2/r)}\right)\exp[i\omega t - kr)]$$

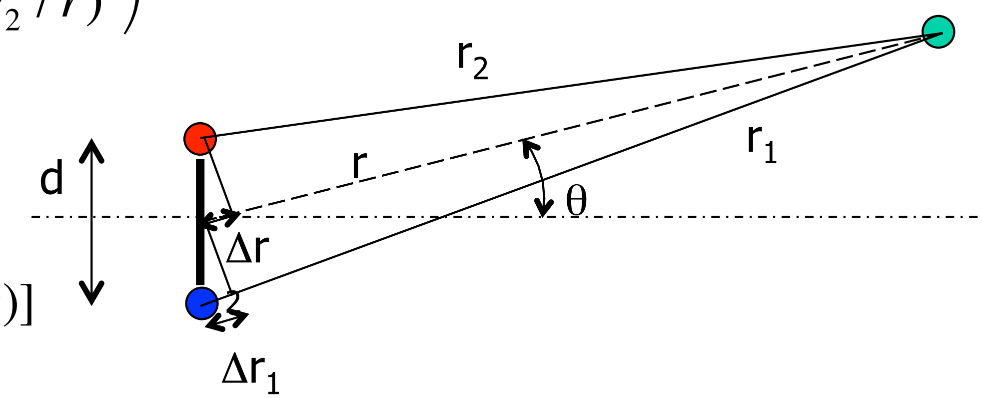
  - Usually interested in r >>d: far-field

$$r >> d \rightarrow \Delta r_1 \approx \Delta r_2 = \frac{d}{2}\sin\theta, \text{ and } \Delta r_i / r << 1$$

$$\text{so } p(r,t) \approx -i\left(\frac{2A}{r}\right)\sin\left(\frac{kd}{2}\sin\theta\right)\exp[i\omega t - kr)]$$

$$\text{for } d << \lambda \rightarrow kd << 1, \ \sin(kd\theta) \approx kd\theta$$

$$p(r,t) \approx -i\left(\frac{Akd}{r}\right)\sin\theta\exp[i\omega t - kr)]$$



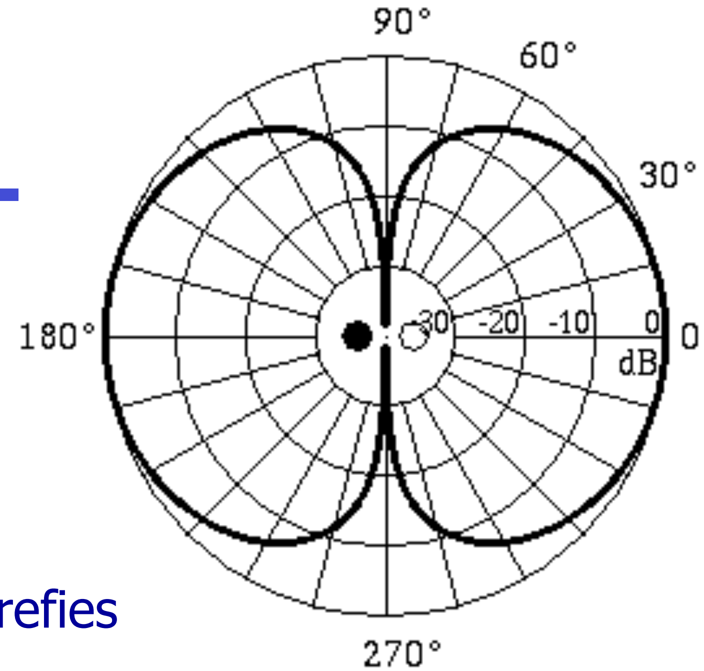
Dipole radiation viewed from the far field

Dipole sources

- Dipoles are commonplace
  - Vibrating tine of a music box
  - Vibrating string or bar
  - Half of a tuning fork

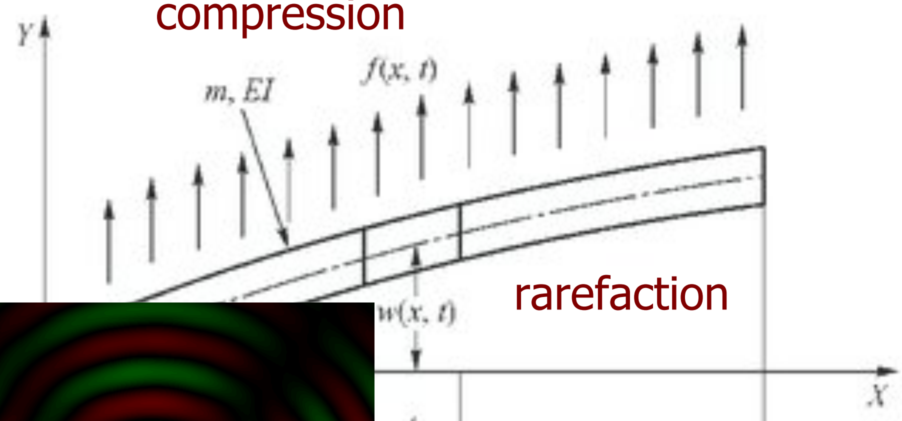
  (full tuning fork = quadrupole source!)

  In each, one side compresses while the other rarefies

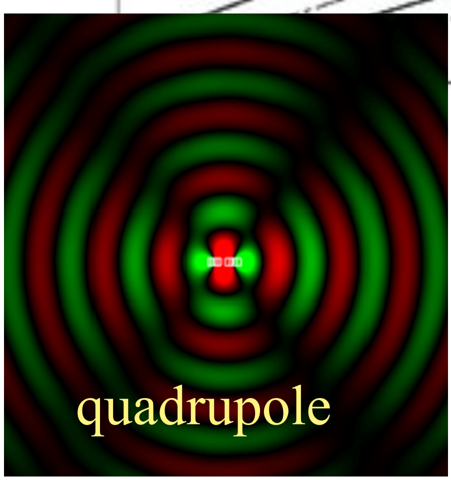


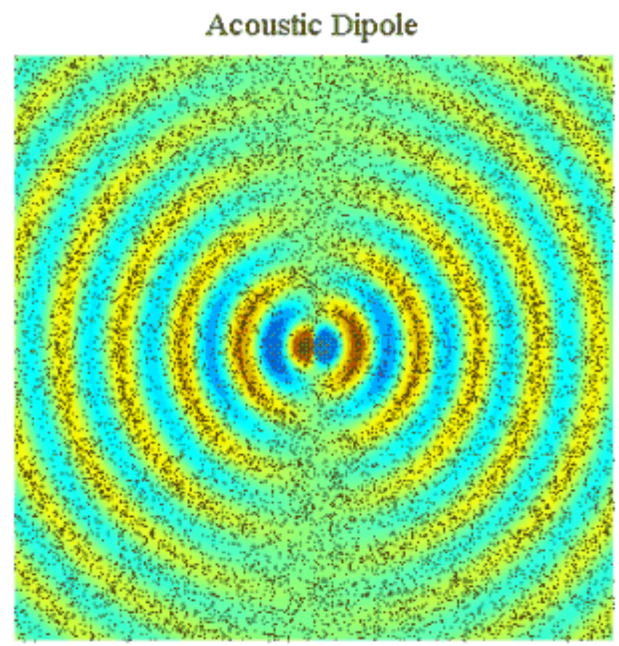
Dipole source radiation patterns

compression



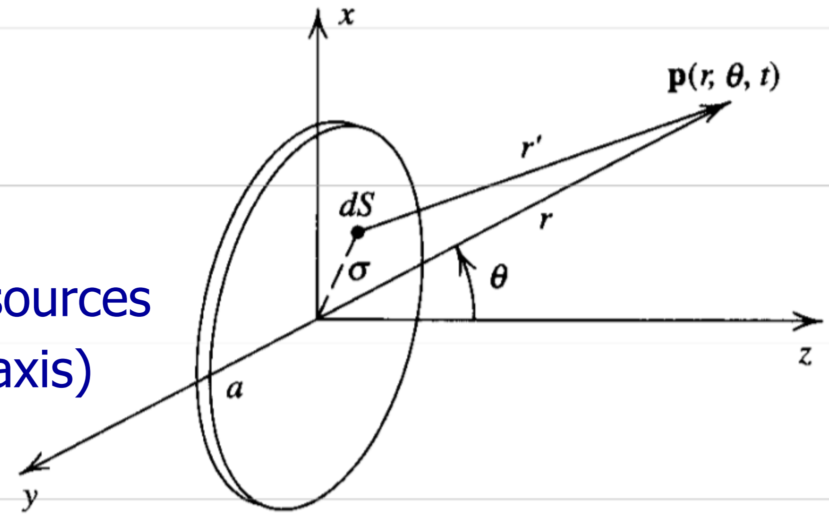
rarefaction





Dipole sources cancel each other in the near field – faint sound. Sounding boards or resonators are needed to hear them.

Plane circular piston source



- Piston is a common model for sources
  - Treat piston as array of baffled simple sources

Start with *p* on the acoustic symmetry axis (z axis)

$$p(r,\theta,t) = i\ \rho_0 c \frac{U_0}{\lambda} \exp(i\omega t) \int_S \frac{1}{r'} e^{i(\omega t - kr')} dS$$

S = surface enclosing piston

For r on the z-axis (θ=0)

$$p(r,\theta=0,t) = i\ \rho_0 c \frac{U_0}{\lambda} \exp(i\omega t) \int_{\sigma=0}^{a} \frac{\exp\left(-ik\sqrt{r^2+\sigma^2}\right)}{\sqrt{r^2+\sigma^2}} 2\pi\sigma\, d\sigma$$

$$\sigma \frac{\exp\left(-ik\sqrt{r^2+\sigma^2}\right)}{\sqrt{r^2+\sigma^2}} = -\frac{d}{d\sigma}\left(\frac{\exp\left(-ik\sqrt{r^2+\sigma^2}\right)}{ik}\right) \qquad \text{where } k = \frac{2\pi}{\lambda}$$

$$\text{so } p(r,\theta=0,t) = i\ \rho_0 c U_0 \frac{2\pi}{\lambda} \exp(i\omega t) \int_{\sigma=0}^{a} d\left(\frac{\exp\left(-ik\sqrt{r^2+\sigma^2}\right)}{ik}\right)$$

$$= \rho_0 c U_0 \left\{1 - \exp\left[-ik\left(\sqrt{r^2+a^2} - r\right)\right]\right\} \exp[i(\omega t - kr')]$$

Plane circular piston source

- The pressure amplitude *P* is the magnitude of *p* :

$$P(r,0) = 2\rho_0 c U_0 \left| \left\{ \sin\left[(1/2)kr\left(\sqrt{1+(a/r)^2} - 1 \right) \right] \right\} \exp[i(\omega t - kr')] \right|$$

If $\left(\frac{r}{a}\right) >> 1$, $\sqrt{1+(a/r)^2} \to 1 + \frac{1}{2}(a/r)^2$

and also $\left(\frac{r}{a}\right) >> \frac{ka}{2} \to r >> \frac{\pi a^2}{\lambda}$,

(piston area)/λ is called the "Rayleigh length"

$$P(r,0) = \frac{1}{2}\rho_0 c U_0 \left(\frac{a}{r}\right) ka$$

$P(r,0) \propto \sin\left[(1/2)kr\left(\sqrt{1+(a/r)^2} - 1 \right) \right] \to$ has maxima and minima

when $(1/2)kr\left(\sqrt{1+(a/r)^2} - 1 \right) = m\pi/2, \;\; m = 0,1,2\ldots$

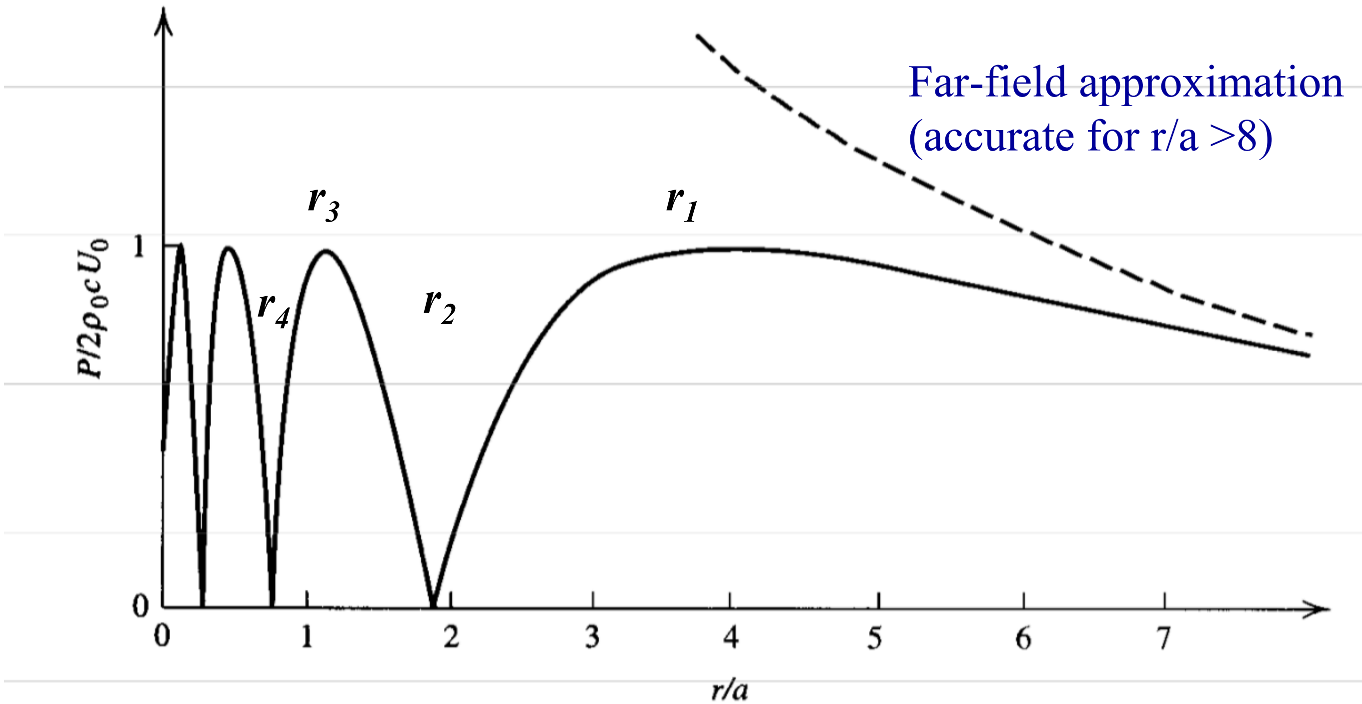
so extrema occur for $r_m / a = (a/m\lambda) - (m\lambda/4a)$

$r_1 / a = (a/\lambda) - (\lambda/4a), \;\; r_2 / a = (a/2\lambda) - (2\lambda/4a),$ etc

until face of piston is reached

Example: on-axis *P* for circular piston

- Coming in from the far field, successive max/mins appear:
  - Pressure amplitude on the z axis for ka=8π → a=4λ



$$r_m / a = (a / m\lambda) - (m\lambda / 4a), \quad m = 1, 2\ldots$$

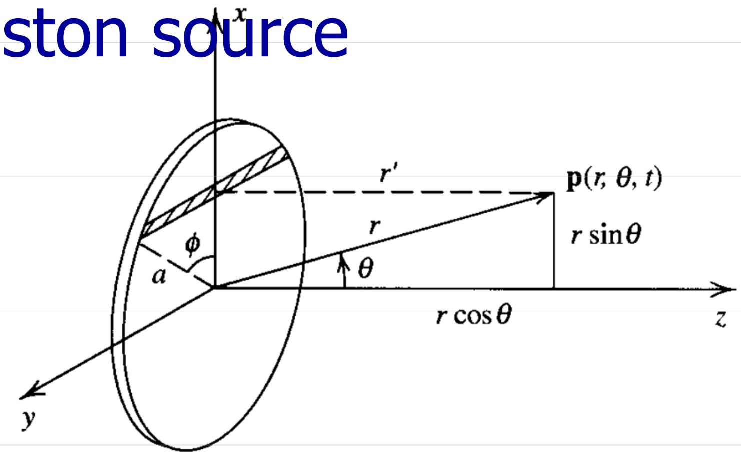
first (max) is $(r_1 / a) = 4 - (1/16)$, next (minimum) at $r_2 / a = (2) - (1/8)$,

max at $r_3 / a = (4/3) - (3/16)$, min at $r_4 / a = (1) - (1/4)$,

until face of piston is reached (here, at $m = 8$)

$r_1 \sim$ beginning of the near field; if $a < \lambda / 2$, $r_1 = 0 \rightarrow$ piston is ~ simple source.

Far field off-axis for plane circular piston source



- Another case where we can integrate *p* is for large r (far field) -- for a point (r,θ) in the x-z plane, we can use strips dx:

$$p(r,\theta,t) = i\ \rho_0 c \frac{U_0}{\lambda} \exp(i\omega t) \int_S \frac{1}{r'} e^{i(\omega t - kr')} dS$$

$$dp = i\ \rho_0 c \frac{U_0}{\pi r'} ka \sin\phi e^{i(\omega t - kr')} dx \quad \text{for } r >> a,$$

$$r' \approx r + \Delta r = r - a\sin\theta\cos\phi$$

(1/r′ ~ 1/r, but must keep Δr in the phase factor)

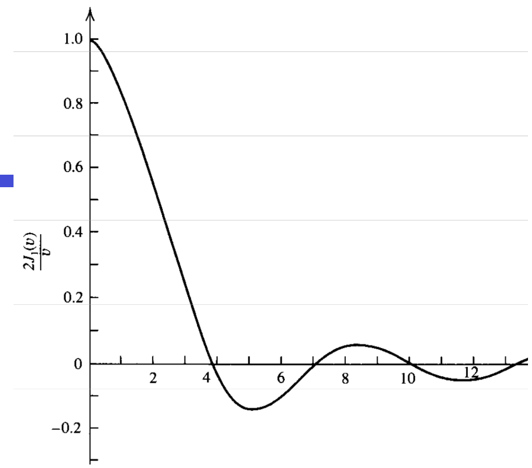
$$p(r,\theta,t) = i\ \rho_0 c \frac{U_0}{\pi r} ka \exp[i(\omega t - kr')] \int_{-a}^{+a} \exp(ika\sin\theta\cos\phi)\sin\phi\, dx;$$

$$x = a\cos\phi \rightarrow dx = a\sin\phi d\phi$$

$$\rightarrow p(r,\theta,t) = i\ \rho_0 c \frac{U_0}{\pi} \left(\frac{a}{r}\right) ka\, e^{i(\omega t - kr)} \int_0^{\pi} \exp(ika\sin\theta\cos\phi)\sin^2\phi\, d\phi$$

$$\mathrm{Re}\left[\int_0^{\pi} \exp(ika\sin\theta\cos\phi)\sin^2\phi\, d\phi\right] = \int_0^{\pi} \cos\left[(ka\sin\theta)\cos\phi\right]\sin^2\phi\, d\phi$$

Far field for plane circular piston source



Integral → Bessel function: $\int_0^{\pi} \cos[z\cos\phi]\sin^2\phi\, d\phi = \pi J_1(z)/z$

so $p(r,\theta,t) = \frac{i}{2}\,\rho_0 c U_0 \left(\frac{a}{r}\right) ka \left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right] e^{i(\omega t - kr)}$

- All the angular dependence is in the square brackets
  - [ ] →1 as θ →0, so we can factorize:

$|p(r,\theta)| = P_{AXIAL}(r)H(\theta)$, where $H(\theta) = \left|\frac{2J_1(v)}{v}\right|$, with $v = ka\sin\theta$

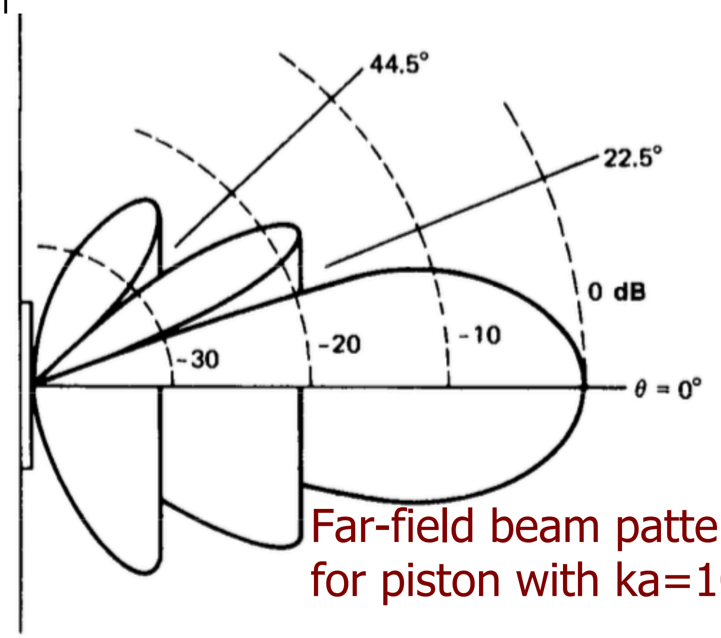
$P_{AXIAL}(r) = \frac{i}{2}\,\rho_0 c U_0 \left(\frac{a}{r}\right) ka$

There is a maximum at $\theta = 0$,

and pressure nodes at angles where $J_1(v) = 0$:

$v_m = ka\sin\theta_m, \quad m = 1,2,3\ldots$

For ka >>1 (λ < a), many narrow lobes;
For ka ~ 1, one broad lobe
For ka << 1 (long λ) → pattern of simple source with strength $Q=\pi a^2 U_0$



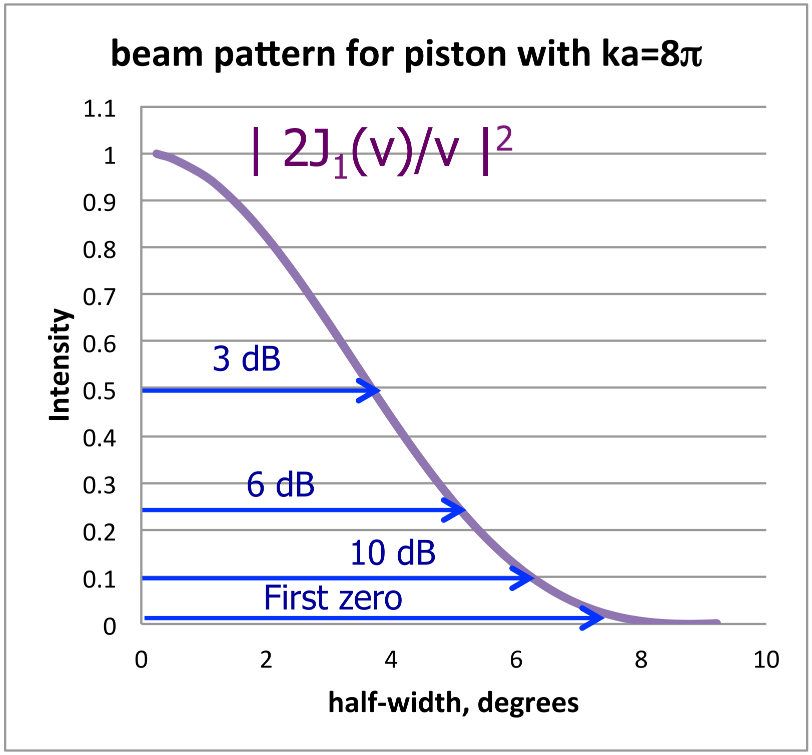
Far-field beam pattern for piston with ka=10

Defining source characteristics

- How to specify the properties of an acoustical source?
  - In many cases there are no agreed standard values – must read carefully!
- Beam width: $\underline{+}\Delta\theta$, $\Delta\phi$ from axis where intensity drops by *x*
  - Factors of $I(\theta, \phi) / I_{AXIS}$=0.25 (6 dB down), 0.5 (3 dB), 10 (10 dB)
  - Example: for piston with ka=8π $\rightarrow$ λ =a/4,
    - 3 dB down = 7.4^{o} , 6 dB down = 10^{o} , 10 dB down = 13^{o} , and the first zero is at 17^{o}
- Source level SL:

  $\langle P \rangle = P_{rms} = P_{axial}/\sqrt{2}$; $SL = 20 \log(P_{rms}/P_{ref})$

  Where (usually) P_{ref} = 1μPa



Defining source characteristics

- Directivity *D* : how concentrated is a source's angular distribution, relative to simple isotropic source?

$$\text{Angular variation of } P(\theta,\phi) = H(\theta) = \left|\frac{2J_1(v)}{v}\right|, \quad \text{with } v = ka\sin\theta$$

$$\text{Intensity} \approx H^2 \rightarrow \text{Directivity } D = \frac{I_{AXIAL}(r)}{I_{SIMPLE}(r)} :$$

intensity of given source / that of simple (spherical pattern) source

$$\frac{I_{AXIAL}(r)}{I_{SIMPLE}(r)} = \frac{P^2_{AXIAL}(r)}{P^2_{SIMPLE}(r)} \rightarrow D = \frac{4\pi}{\left\langle H^2(\theta)\right\rangle_\Omega} ,$$

where $\left\langle H^2(\theta)\right\rangle_\Omega$ = average over solid angle Ω

  - Directivity index: *DI = 10 log D*

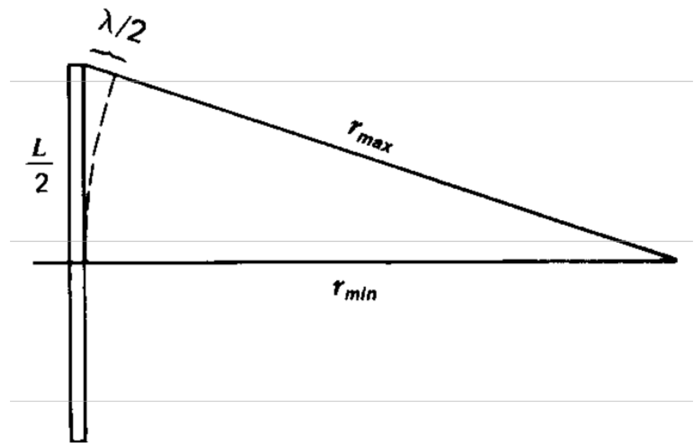
Far field

- Where is it safe to use far-field approximation?
  - A source is directive if $kL >> 1 \rightarrow \lambda << 2\pi L$, L = size of source
  - One definition of where FF begins is: for a point on axis,

r_{MAX} = distance to edge of source,

r_{MIN} = distance to center of source

FF begins where $(r_{MAX} - r_{MIN}) \sim \lambda/2$

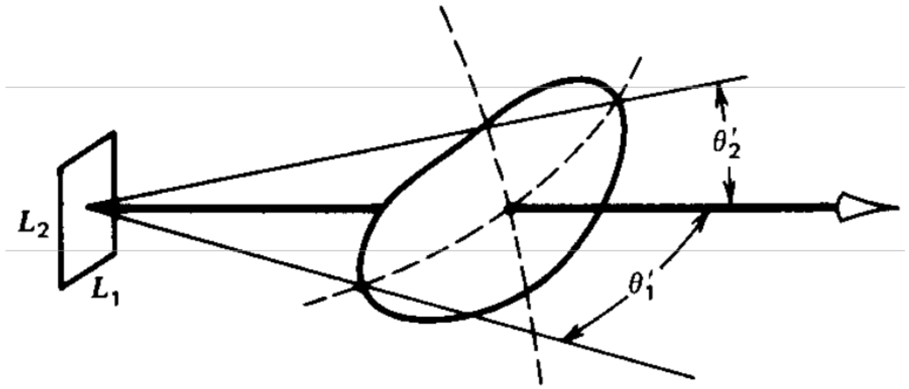


- Estimating major lobe width
  - First minimum (defining the "major lobe") is where lower half of piston is out of phase with upper half

$$L\sin\theta_1 \approx \lambda \rightarrow \sin^{-1}\theta_1 \approx \frac{\lambda}{L}$$

Diagram shows case where source is rectangular, L_1 x L_2

(r_{MIN} for FF is determined by the larger of L_1 and L_2)



Radiation impedance

- We defined input mechanical impedance for a driven string:

  $Z_m = f_{DRIVER} / u(x=0,t)$ (f_{DRIVER} = force at driven end)

  - For sources, we can similarly define radiation impedance as

$$Z_r = \int_{AREA} \frac{df}{u}, \text{ (integral is over active face area, } df \text{ may vary)}$$

$Z_r = R_r + iX_r$ radiation resistance and reactance*

If source face is rigid, has mass m, stiffness ("spring constant") s,

with driving force $f(t) = F\exp(i\omega t)$ producing face speed $u_0 = U_0 \exp(i\omega t)$,

$$\text{then Newton} \rightarrow F_{net} = ma \rightarrow F - R_m \frac{dx}{dt} - sx = m\frac{d^2x}{dt^2}$$

$$\rightarrow \left(f - f_s\right) - R_m \frac{d\xi_0}{dt} - s\xi_0 = m\frac{d^2\xi_0}{dt^2},$$

where fluid's reaction force on face is $f_s = Z_r u_0$

and ξ_0 = particle displacement at face of source.

Recall (Kinsler Ch. 1) $Z_m = R_m + iX_m$,

$$X_m = \omega m - s/\omega \rightarrow U_0 = \frac{F}{Z_m + Z_r}$$

Source sees Z_{EFF} = its own Z_m + fluid's radiation impedance

*Total power radiated is

$$\mathbf{P}_{TOT} = \frac{1}{T}\int_0^T \mathrm{Re}\left[f_s \cdot u_0\right] dt$$

Find *R* from power dissipated:

$$\mathbf{P}_{TOT} = \frac{1}{2} U_0^2 R_r$$

Reactance does not dissipate P, acts like extra mass load:

$m \rightarrow m + m_r$ where $m_r = X_r / \omega$

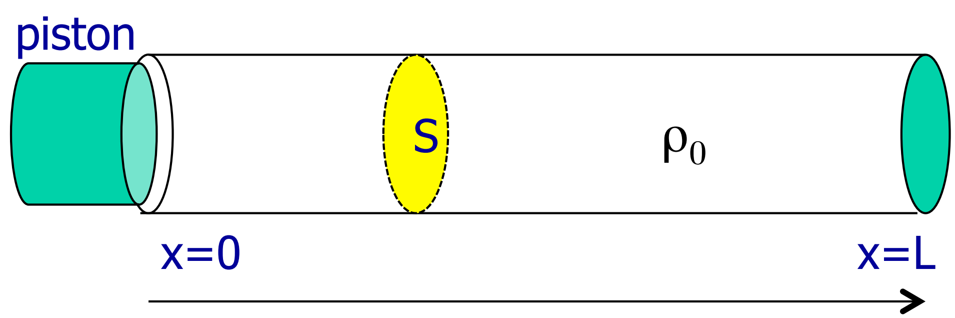
so $\omega_0 = \sqrt{s/m} \rightarrow \sqrt{s/(m + m_r)}$

= lowered resonant frequency

Waves in pipes

- Waves in pipes are important for musical instruments (we will get to them later), ventilation ducts, and other applications
  - In a rigid-walled pipe with R << λ , sound propagates as plane waves, similar to longitudinal waves in solid bars
  - For rigid-walled volumes with R > λ , standing waves can appear: resonances
- Example: pipe has cross-sectional area *S*, length *L*, filled by fluid with density ρ_0 , and driven by a piston at *x=0* ; end at *x=L* is closed by a plug with mechanical impedance Z_{mL}
  - As usual we get

$$p(x,t) = Ae^{i[\omega t+k(L-x)]} + Be^{i[\omega t-k(L-x)]}$$
$$= p_+ + p_-$$



Continuity of force, particle speed $\rightarrow$
must have impedance match: $Z_{mL} = Z_{wave}$

$$f(L,t) = p(L,t)S; \;\; Z_{wave} = f(L,t)/u(L,t)$$

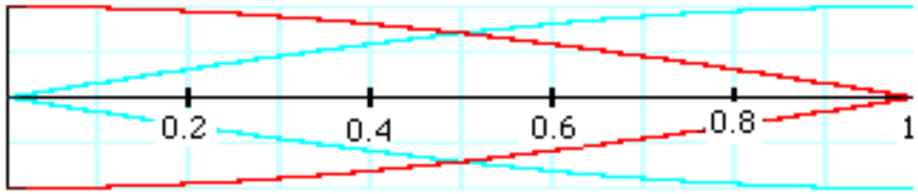
Driven closed-end pipe resonant f's

$$\rightarrow \quad Z_{mL} = (\rho_0 c) S \frac{A+B}{A-B};$$

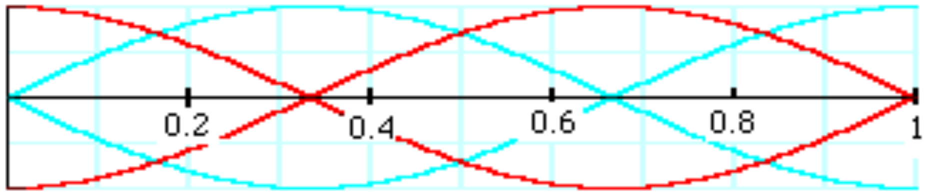
$$Z_{m0} = (\rho_0 c) S \frac{Ae^{ikL} + Be^{-ikL}}{Ae^{ikL} - Be^{-ikL}}$$

solve to eliminate A, B:

$$\frac{Z_{m0}}{\rho_0 c S} = \frac{\dfrac{Z_{mL}}{\rho_0 c S} + i \tan kL}{1 + \dfrac{Z_{mL}}{\rho_0 c S} i \tan kL}$$



(even harmonics are absent)



For a rigid end cap, Z_{mL} is very large, so

$$\frac{Z_{m0}}{\rho_0 c S} = \frac{r + ix}{\rho_0 c S} \approx \frac{1}{i \tan kL} = -i \cot(kL)$$

$r \approx 0$ and $x = 0$ when $\cot(kL) = 0$

$$\rightarrow \quad k_n L = (2n-1)(\pi/2), \quad n = 1, 2, 3 \ldots$$

$$\rightarrow \quad f_n = \frac{c k_n}{2\pi} = \frac{(2n-1)}{4} \frac{c}{L}$$

Driven closed pipe has a pressure node at x=0, and antinode at x=L (opposite for u)

Driven open-end pipe resonant f's

- In intro physics, we take open end of pipe to have $Z_{mL} = 0$
- Actually, open end sees Z of room air: $Z_{mL} = Z_r$
- An open pipe with a large flange (eg, air duct in wall) is similar to piston in a baffle, so

$$\frac{Z_{mL}}{\rho_0 c S} \approx r + ix = \frac{1}{2}(ka)^2 + i\frac{8(ka)}{3\pi}$$

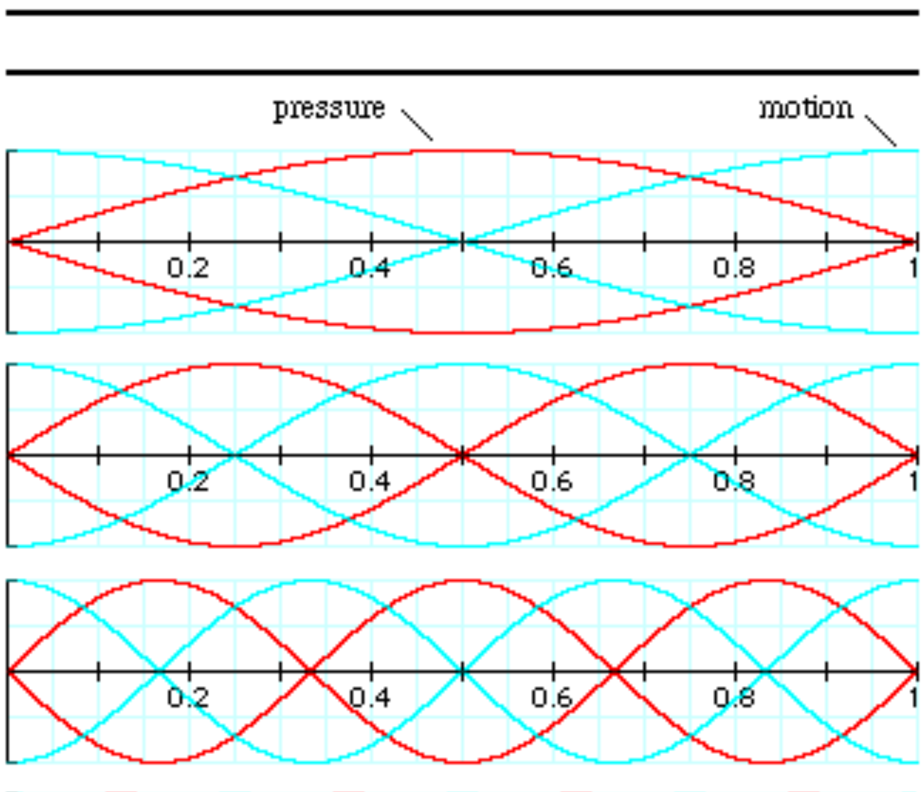
where typically $r << x << 1$

Resonant frequencies occur when

$$k_n L + \frac{8}{3\pi} k_n a = n\pi, \quad n = 1,2,3...$$

$$\rightarrow f_n = \frac{n}{2}\frac{c}{L_{EFF}}, \quad L_{EFF} = L + \frac{8}{3\pi} a$$

For an unflanged pipe, $L_{EFF} = L + (0.6)a$



Ideal driven open pipe has a pressure node at x=0, and antinode at x=L (opposite for u).

Real pipe needs end correction: **Effective length > L**

Power radiated from open-ended pipes

- For open ended pipes, we had $p(x,t) = Ae^{i[\omega t + k(L-x)]} + Be^{i[\omega t - k(L-x)]}$

$$= p_+ + p_+$$

$$\rightarrow \frac{Z_{mL}}{\rho_0 cS} = \frac{A+B}{A-B} \rightarrow \frac{B}{A} = \frac{Z_{mL}/\rho_0 cS - 1}{Z_{mL}/\rho_0 cS + 1}$$

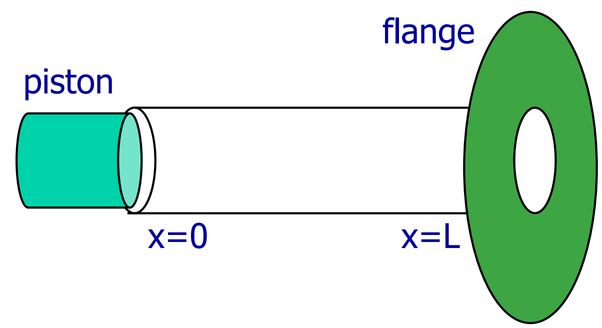
$B/A =$ *amplitude* reflection coefficient

Power reflection coeff $= |B/A|^2 \rightarrow T = 1 - |B/A|^2$

T = transmission coefficient for power out of pipe

Results: for flanged open end, $T \approx 2(ka)^2 << 1$

$T << 1 \rightarrow |B/A|^2 \approx 1$; actually, $B/A \approx -1$ (see Kinsler ch. 9)



- So $p_- \sim p_+$, and at x=L reflected wave is phase-flipped
- Particle velocities are in phase → end is antinode of speed
- For unflanged pipe, $T=(ka)^2$ → wide flange doubles output
  - Flare at end instead of flange increases power output (eg, trumpet)
- Only small fraction of power is transmitted out of pipe
  - Characteristic of sources small relative to wavelength

Standing waves in pipes

- For sound waves in a pipe, wave pressure is

$$\mathbf{p}(x,t) = \mathbf{A}e^{i[\omega t + k(L-x)]} + \mathbf{B}e^{i[\omega t - k(L-x)]} = p_+ + p_-$$

allowing for a phase difference between p_- and p_+: $\mathbf{A} = A, \quad \mathbf{B} = Be^{i\theta}$

we find $\dfrac{B}{A} = \dfrac{Z_{mL}/\rho_0 cS - 1}{Z_{mL}/\rho_0 cS + 1} \rightarrow \dfrac{Z_{mL}}{\rho_0 cS} = \dfrac{1+(B/A)e^{i\theta}}{1-(B/A)e^{i\theta}}$

$$\rightarrow |p| = \left\{ \left(A+B\right)^2 \cos^2\left[k(L-x)-\theta/2\right] + \left(A-B\right)^2 \sin^2\left[k(L-x)-\theta/2\right] \right\}$$

→ Pressure amplitude at a node = (A+B), at antinode = (A-B)

- Define Standing Wave Ratio as SWR = p(node) / p(antinode)

  SWR =(A+B)/(A-B) → (B/A)=(SWR-1) / (SWR+1)

  - Can find SWR by measuring sound intensity in a pipe: move microphone from L downward

  SWR = max amplitude/min amplitude

  find phase angle from location of first node

  Nodes have $k(L-x)-\theta/2 = \left(n-1/2\right)\pi \rightarrow \theta_1 = 2K\left(L-x_1\right)-\pi$

Finding Z_m from SWR measurements

- Example: For sound waves in a pipe with rigid endcap, measurements give SWR = 2, and x_1 =(3/8)λ from L
  - Then

$$\theta_1 = 2k\left(L - x_1\right) - \pi = 2\left(\frac{2\pi}{\lambda}\right)\left(L - \left(\frac{3\lambda}{8}\right)\right) = \frac{\pi}{2}$$

$$\text{we find}\quad \frac{B}{A} = \frac{(SWR - 1)}{(SWR + 1)} = \frac{(2 - 1)}{(2 + 1)} = \frac{1}{3}$$

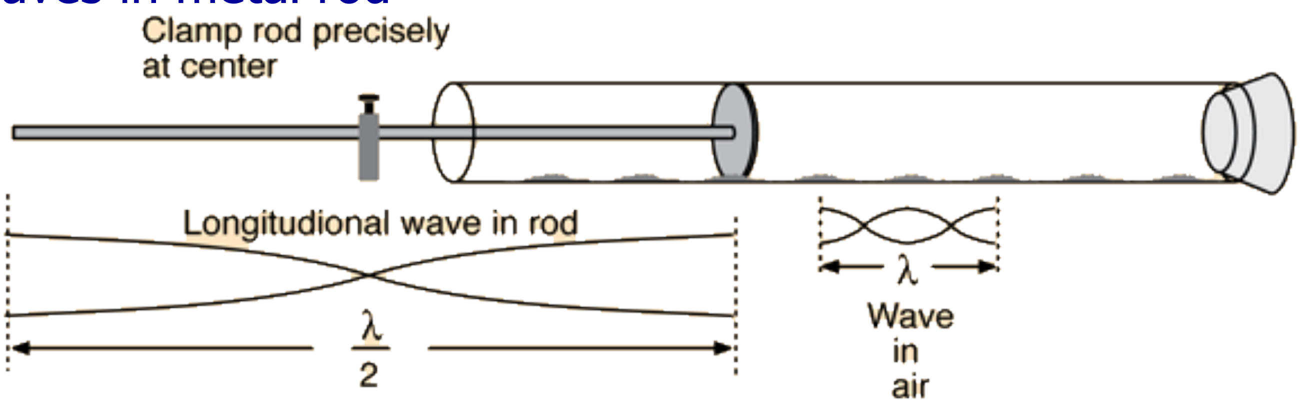
$$\rightarrow \frac{Z_{mL}}{\rho_0 c S} = \frac{1 + (B/A)e^{i\theta}}{1 - (B/A)e^{i\theta}} = \frac{1 + e^{i\pi/2}/3}{1 - e^{i\pi/2}/3} = 0.80 + i\,(0.60)$$

- Impedance may depend on frequency – repeat measurements
- Tool for matching impedance = Smith Chart
  - Study the locus of a component's R and X versus frequency: see digikey.com/en/articles/techzone/2014/mar/the-smith-chart-an-ancient-graphical-tool-still-vital-in-rf-design
- Can visualize locations of nodes with a Kundt Tube

Kundt's tube

demo: https://www.youtube.com/watch?v=qUiB_zd9M0k

- Same idea as Chladni plates, applied to cylindrical tube
  - August Kundt (1866): measured speed of sound in gases and metals
  - Set up standing waves in glass cylinder filled with fine powder
    - Powder accumulates at nodes: node spacing = λ / 2
  - 19th C: set up wave of known λ by stroking metal rod on disk
    - Clamped at center → node → rod length = λ / 2 for longitudinal waves in metal rod



  - Today: instead of passive end plug and stroked rod, use loudspeaker and signal generator

particle displacement $\xi(t)=0$ at passive end, $\xi = a\cos(\omega t)$ at speaker

$$\xi(t) = A\sin\left(\frac{\omega}{c}x\right)\cos(\omega t) \text{ so nodes occur where } x = \frac{c}{2\pi f}n\pi,\quad n = 0,1,2\ldots$$

Kundt Tube Dust Striations

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(Received January 10, 1955)

An interesting anomaly in the familiar Kundt tube experiment and the eighty-year search for an interpretation is reviewed for teachers of physics. Unfortunately the correct explanation of this phenomenon, although available, is rarely cited and known to very few. The work and explanation of Andrade are presented and expanded in hopes of remedying this situation.

MOST teachers of physics are familiar with the Kundt tube[1] apparatus for the demonstration and investigation of standing waves in air; but a little questioning will show that very few know what to say when a student asks why the dust figures in the tube are striated (see Fig. 1). A guess most often heard is that the vibration of the rod evidently emphasizes a single high harmonic, and this harmonic produces the ripples just as the fundamental produces the larger pattern. But this is incorrect. The search for a true explanation of the striated

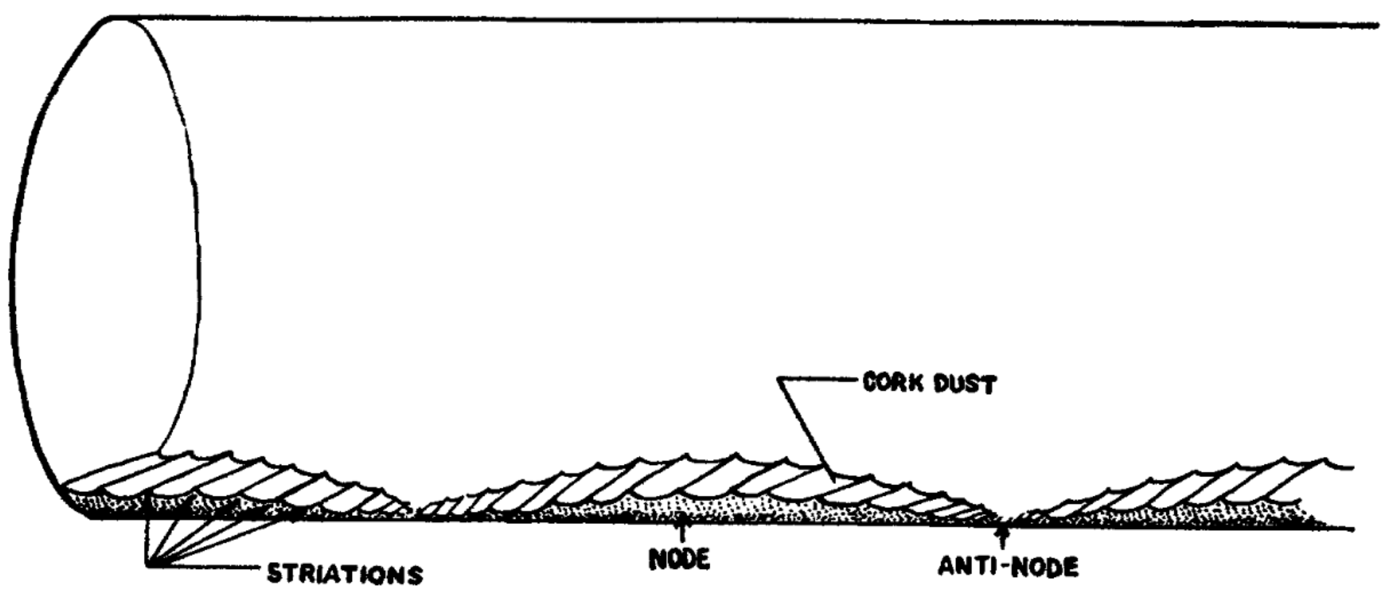

FIG. 1. Schematic diagram of Kundt tube showing striations.

Kundt's tube puzzle: what causes the striations?

- K's tube demo is performed everywhere, but few can answer "what are those closely spaced little columns of powder in the standing wave peaks?"!
  - Wrong answer: higher overtones in the tube (λ too small – need huge n)
- Answer: vortex formation due to failure of assumption that flow in tube is laminar
  - For full explanation, *see Kundt Tube Dust Striations*, Carman, Robert A., American journal of physics, 1955, Vol.23, p.505-507

Photographic studies by Andrade show that any particle suspended in the medium which does not share in the vibrational motion of the air column may become the center of a localized vortex motion (Fig. 2). Thus at proper conditions a pattern of such vortices will exist in the tube. A glance at

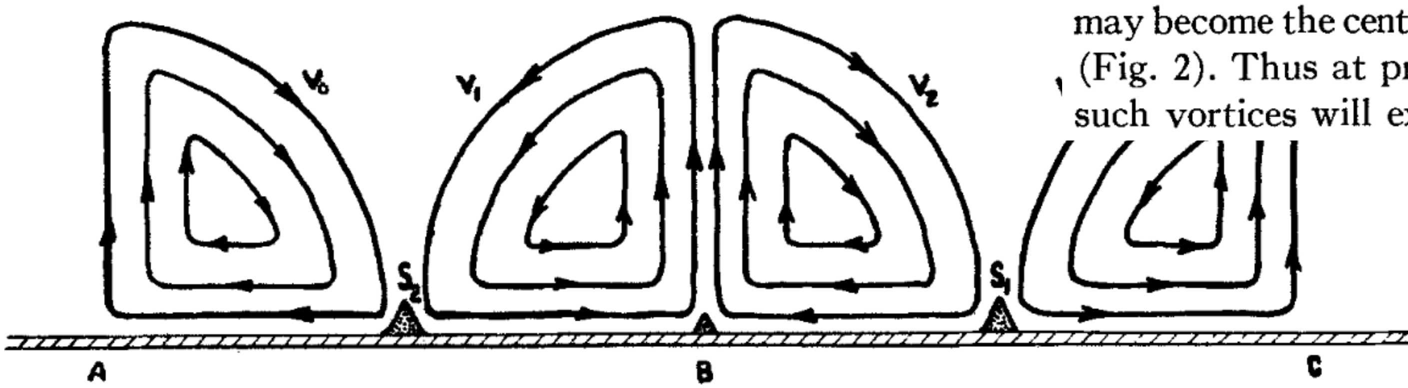

FIG. 3. Cross section of Kundt tube showing the ripple formation process. Dust is swept up at points A and B by vortices V_0 and V_1 to form a ripple at S_2. Similarly a ripple at S_1 is formed by vortices V_2 and V_3. Minor or secondary ripples are formed by the remaining dust at points A, B, and C.