PHYS 536

R. J. Wilkes

Session 9

Radiation impedance

Kundt’s tube

Helmholtz resonators

Musical acoustics

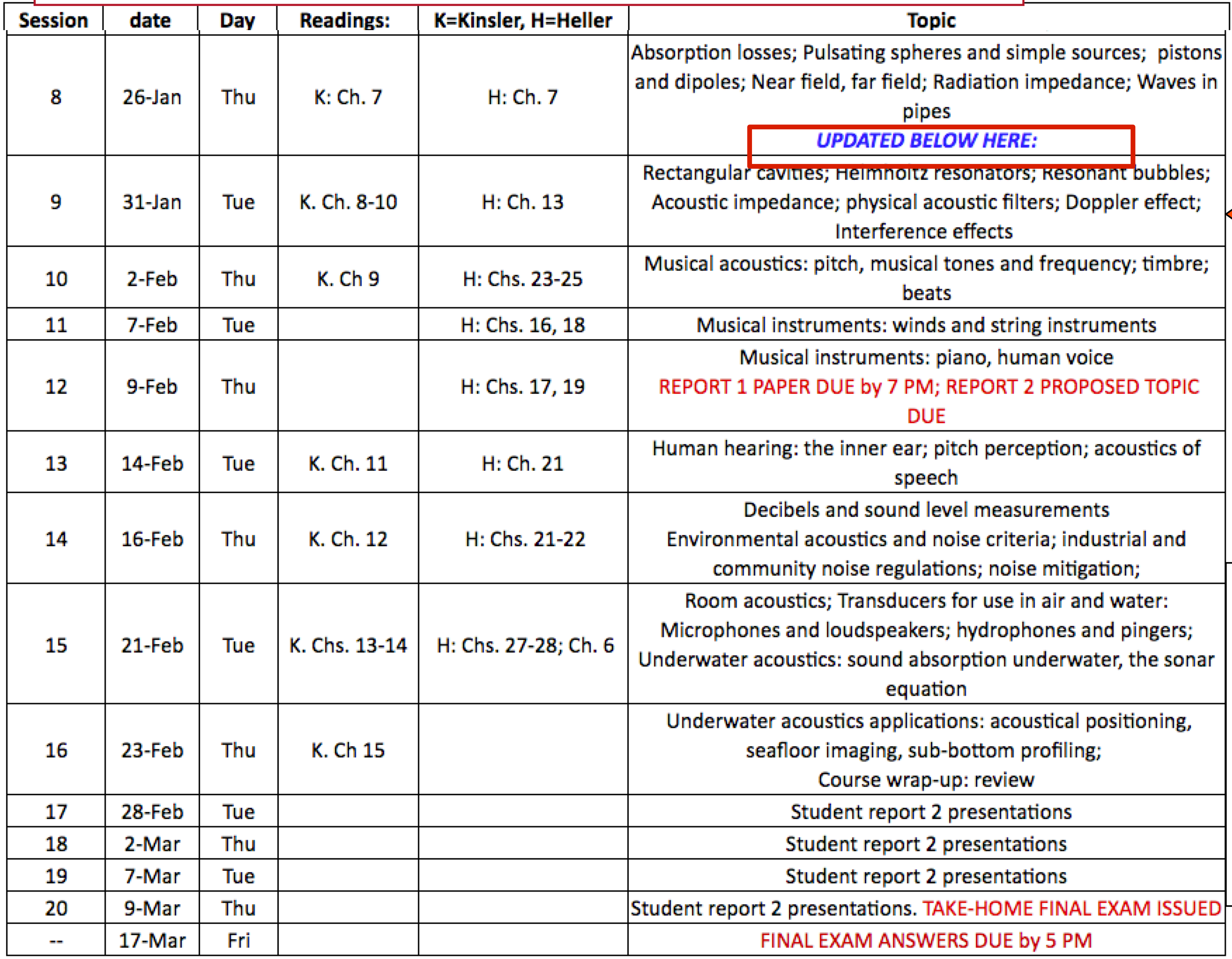
1/31/2023

Announcements

- Schedule of topics has been rearranged – pls check readings
- Due this Thursday!
  - Papers for project 1
  - Proposals for project 2 presentation

Course syllabus and schedule – updated

See : http://courses.washington.edu/phys536/syllabus.htm



Session	date	Day	Readings:	K=Kinsler, H=Heller	Topic
8	26-Jan	Thu	K: Ch. 7	H: Ch. 7	Absorption losses; Pulsating spheres and simple sources; pistons and dipoles; Near field, far field; Radiation impedance; Waves in pipes *UPDATED BELOW HERE:*
9	31-Jan	Tue	K. Ch. 8-10	H: Ch. 13	Rectangular cavities; Helmholtz resonators; Resonant bubbles; Acoustic impedance; physical acoustic filters; Doppler effect; Interference effects
10	2-Feb	Thu	K. Ch 9	H: Chs. 23-25	Musical acoustics: pitch, musical tones and frequency; timbre; beats
11	7-Feb	Tue		H: Chs. 16, 18	Musical instruments: winds and string instruments
12	9-Feb	Thu		H: Chs. 17, 19	Musical instruments: piano, human voice REPORT 1 PAPER DUE by 7 PM; REPORT 2 PROPOSED TOPIC DUE
13	14-Feb	Tue	K. Ch. 11	H: Ch. 21	Human hearing: the inner ear; pitch perception; acoustics of speech
14	16-Feb	Thu	K. Ch. 12	H: Chs. 21-22	Decibels and sound level measurements Environmental acoustics and noise criteria; industrial and community noise regulations; noise mitigation;
15	21-Feb	Tue	K. Chs. 13-14	H: Chs. 27-28; Ch. 6	Room acoustics; Transducers for use in air and water: Microphones and loudspeakers; hydrophones and pingers; Underwater acoustics: sound absorption underwater, the sonar equation
16	23-Feb	Thu	K. Ch 15		Underwater acoustics applications: acoustical positioning, seafloor imaging, sub-bottom profiling; Course wrap-up: review
17	28-Feb	Tue			Student report 2 presentations
18	2-Mar	Thu			Student report 2 presentations
19	7-Mar	Tue			Student report 2 presentations
20	9-Mar	Thu			Student report 2 presentations. TAKE-HOME FINAL EXAM ISSUED
--	17-Mar	Fri			FINAL EXAM ANSWERS DUE by 5 PM

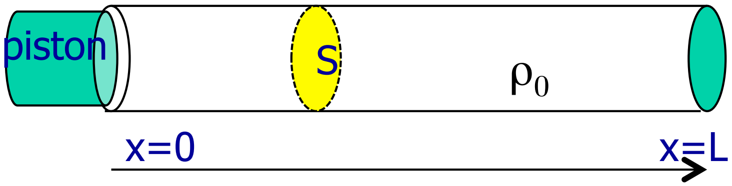
Tonight

Class is over after you turn in your take-home exam. No in-person final exam during finals week.

Last time
(but edited/corrected!)

Closed-end pipe resonant f's

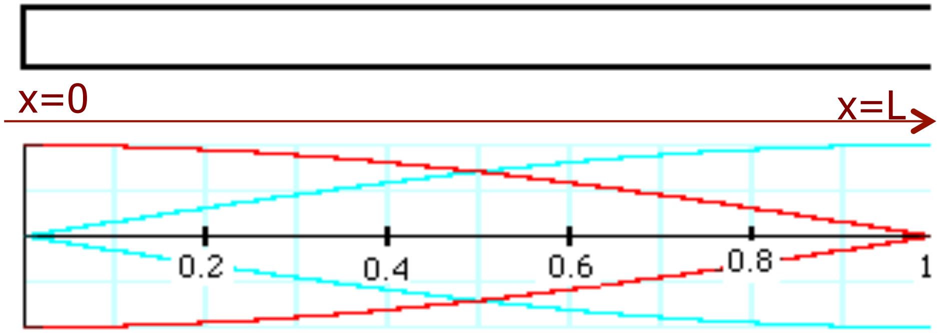
- Driven closed end pipe:



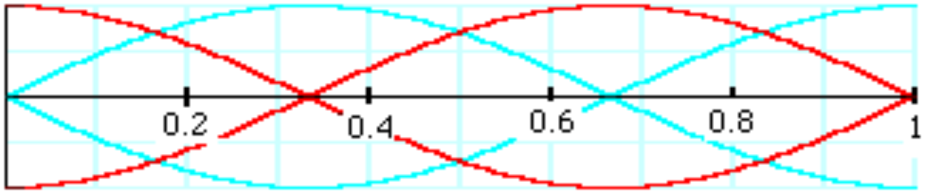
$$p(x,t) = Ae^{i[\omega t + k(L-x)]} + Be^{i[\omega t - k(L-x)]}$$
$$= p_+ + p_-$$

Continuity of force, particle speed $\rightarrow$ must have impedance match: $Z_{mL} = Z_{wave}$

$$f(L,t) = p(L,t)S;\ \ Z_{wave} = f(L,t)/u(L,t)$$



(even harmonics are absent)



- Open end/closed end

At a rigid end cap, Z_{mL} is very large, so

$$\frac{Z_{m0}}{\rho_0 cS} = \frac{r+ix}{\rho_0 cS} \approx \frac{1}{i\tan kL} = -i\cot(kL)$$

$$r \approx 0 \quad \text{and} \quad x = 0 \quad \text{when} \quad \cot(kL) = 0$$

$$\rightarrow\ k_n L = (2n-1)(\pi/2), \quad n = 1,2,3\ldots$$

$$\rightarrow\ f_n = \frac{ck_n}{2\pi} = \frac{(2n-1)}{4}\frac{c}{L}$$

Driven closed pipe has a displacement node at x=0, and antinode at x=L (opposite for p)

Corrected!

Last time
(but edited/corrected

Driven open-end pipe resonant f's

- In intro physics, we take open end of pipe to have $Z_{mL} = 0$
- Actually, open end sees Z of room air: $Z_{mL} = Z_r$
- An open pipe with a large flange (eg, air duct in wall) is similar to piston in a baffle, so

$$\frac{Z_{mL}}{\rho_0 c S} \approx r + ix = \frac{1}{2}(ka)^2 + i\frac{8(ka)}{3\pi}$$

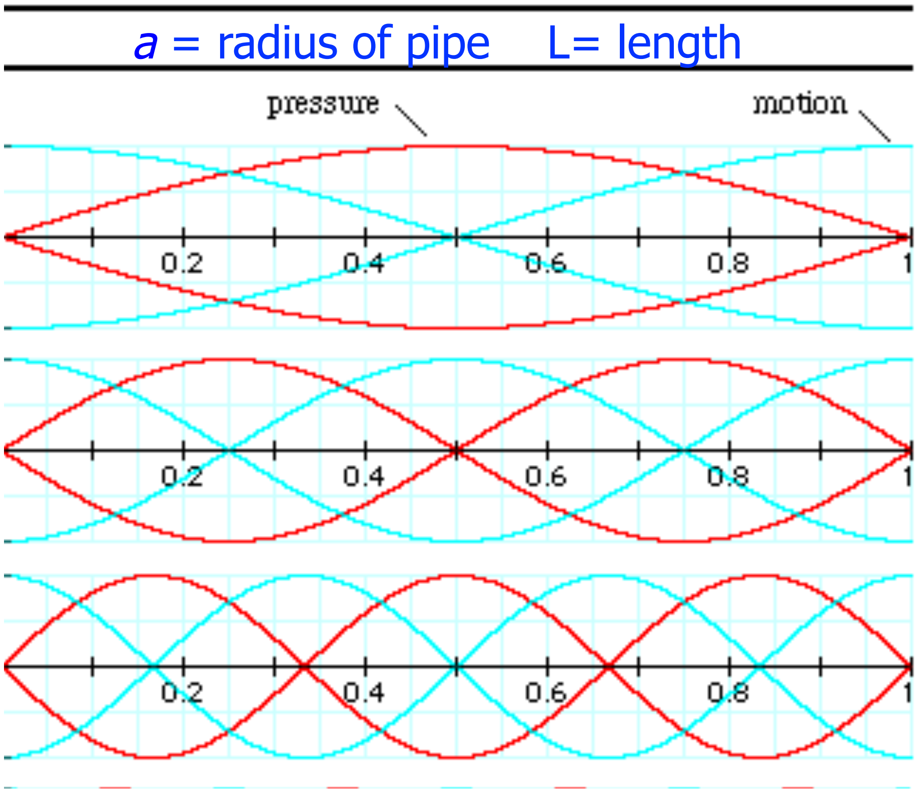
where typically $r << x << 1$

Resonant frequencies occur when

$$k_n L + \frac{8}{3\pi} k_n a = n\pi, \quad n = 1,2,3\ldots$$

$$\rightarrow f_n = \frac{n}{2}\frac{c}{L_{EFF}}, \quad L_{EFF} = L + \frac{8}{3\pi} a$$

For an unflanged pipe, $L_{EFF} = L + (0.6)a$



a = radius of pipe L= length

Ideal driven open pipe has a pressure nodes at x=0 and x=L (opposite for u).

Corrected!

Real pipe needs end correction:
Effective length > L

Power radiated from open-ended pipes

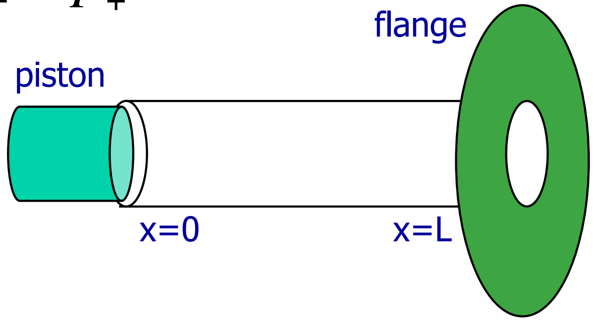
From last time

- For open ended pipes, we had

$$p(x,t) = Ae^{i[\omega t+k(L-x)]} + Be^{i[\omega t-k(L-x)]}$$

$$= p_+ + p_+$$

$$\rightarrow \frac{Z_{mL}}{\rho_0 cS} = \frac{A+B}{A-B} \rightarrow \frac{B}{A} = \frac{Z_{mL}/\rho_0 cS - 1}{Z_{mL}/\rho_0 cS + 1}$$



$B/A =$ *amplitude* reflection coefficient

Power reflection coeff $= |B/A|^2 \rightarrow T = 1 - |B/A|^2$

T = transmission coefficient for power out of pipe

Results: plain open end, $T \approx (ka)^2 << 1$; but for flanged open end, $T \approx 2(ka)^2$

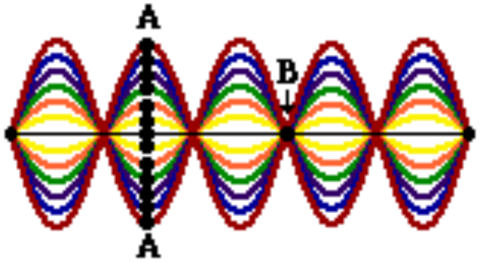
$T << 1 \rightarrow |B/A|^2 \approx 1$; actually, $B/A \approx -1$ (see Kinsler ch. 9)

doubles output !

- So p_ ~ p₊ , and at x=L reflected wave is phase-flipped
- Particle velocities are in phase → end is antinode of speed
- For unflanged pipe, $T=(ka)^2$ → wide flange doubles output
  - Flare at end instead of flange increases power output (eg, trumpet)
- Only small fraction of power is transmitted out of pipe
  - Characteristic of sources small relative to wavelength

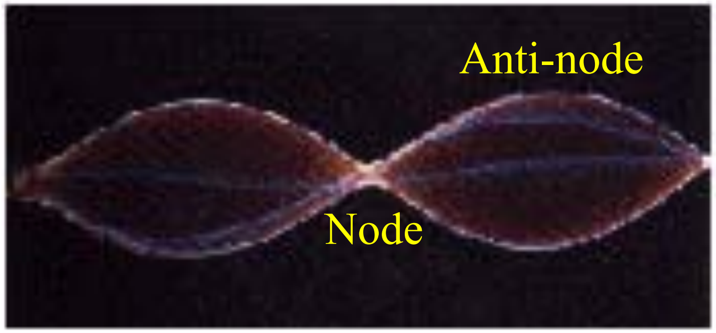
Recall: Self-interference → standing waves

- If we wiggle a rope at *just* the right f
  - Waves reflected from the end *interfere constructively* with new waves I am making
  - Result: looks as if some points stand still: standing waves
    - Example of *resonance:* rope length L = multiple of $\lambda/2$



Point A moves with big amplitude

Point B has amplitude ~0



Nodes = stationary points; anti-nodes=maxima

- Same thing happens in *musical instruments*
  - Structure favors waves which have L = multiple of $\lambda/2$
    - Guitar, violin strings: both ends must be *nodes*

  - Organ pipes, and other wind instruments with one closed end:

one end must be node, other antinode

Interference → standing waves

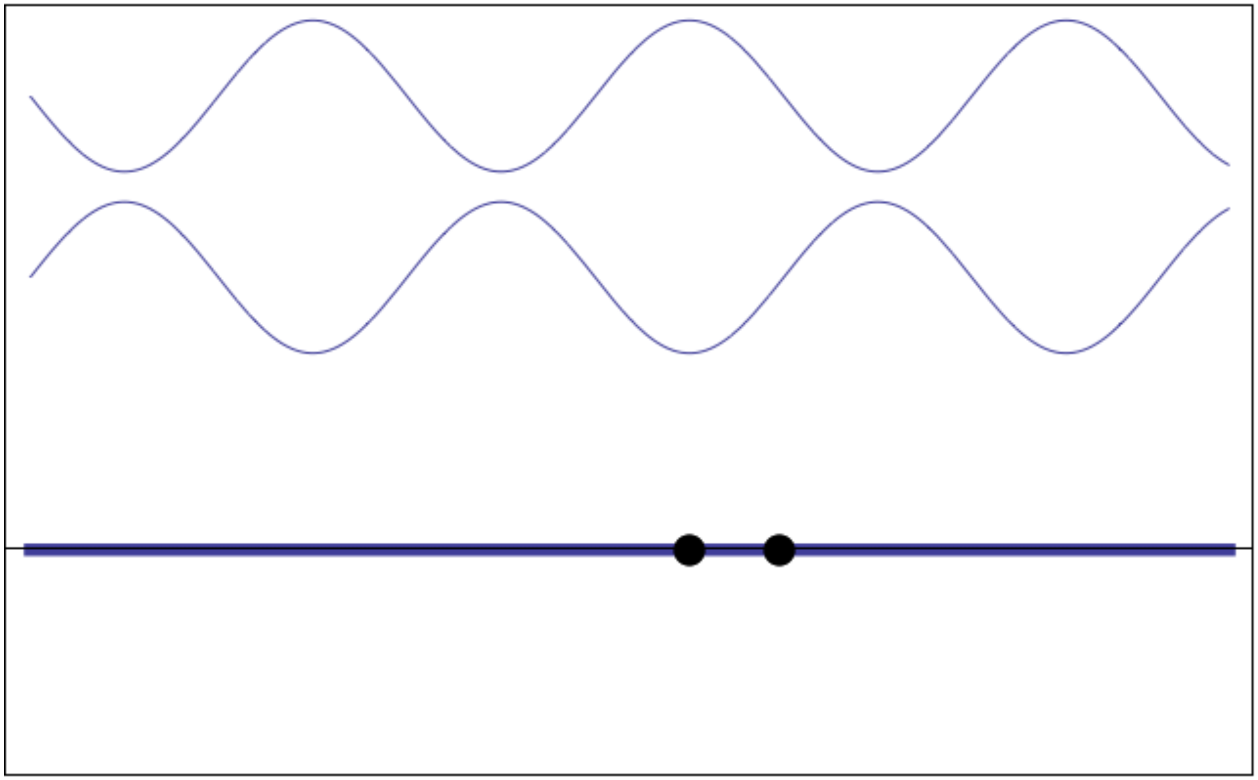
- Two waves propagating in opposite directions with same λ and amplitude superpose to form a standing wave

$$y(x,t) = A\sin(kx-\omega t) + A\sin(kx+\omega t) = 2A\sin(kx)\cos(\omega t)$$

Forward wave   Backward wave   **Trig identity**   Standing wave

Notice:

Amplitude vs x is fixed, but at each x position, y vs t oscillates



Where sin(kx) =0:

Minima = nodes

Where sin(kx) = 1:

Maxima = antinodes

Standing wave ratio SWR

- For sound waves in a pipe, wave pressure is

$$\mathbf{p}(x,t) = \mathbf{A}e^{i[\omega t + k(L-x)]} + \mathbf{B}e^{i[\omega t - k(L-x)]} = p_+ + p_-$$

allowing for a phase difference θ between p_- and p_+: $\mathbf{A} = A, \quad \mathbf{B} = Be^{i\theta}$

we find $\dfrac{B}{A} = \dfrac{Z_{mL}/\rho_0 cS - 1}{Z_{mL}/\rho_0 cS + 1} \rightarrow \dfrac{Z_{mL}}{\rho_0 cS} = \dfrac{1+(B/A)e^{i\theta}}{1-(B/A)e^{i\theta}}$

$$\rightarrow |p| = \left\{ \left(A+B\right)^2 \cos^2\left[k(L-x) - \theta/2\right] + \left(A-B\right)^2 \sin^2\left[k(L-x) - \theta/2\right] \right\}$$

→ Pressure amplitude at a node = (A+B), at antinode = (A-B)

- Define Standing Wave Ratio as SWR = *p(node) / p(antinode)*

  SWR =(A+B)/(A-B) ←→ (B/A)=(SWR-1) / (SWR+1)

  - Can find SWR by measuring sound intensity in a pipe:

  move microphone from L downward

  SWR = max amplitude/min amplitude

  find phase shift from location of first node

  Nodes have $k(L-x) - \theta/2 = \left(n - 1/2\right)\pi \rightarrow \theta_1 = 2k\left(L - x_1\right) - \pi$

Finding Z_m from SWR measurements

- Example: For sound waves in a pipe with rigid endcap, measurements give SWR = 2, and x_1 =(3/8)λ from L
  - Then

$$\theta_1 = 2k\left(L - x_1\right) - \pi = 2\left(\frac{2\pi}{\lambda}\right)\left(L - \left(\frac{3\lambda}{8}\right)\right) = \frac{\pi}{2}$$

$$\text{we find} \quad \frac{B}{A} = \frac{(SWR - 1)}{(SWR + 1)} = \frac{(2 - 1)}{(2 + 1)} = \frac{1}{3}$$

$$\to \frac{Z_{mL}}{\rho_0 c S} = \frac{1 + (B/A)e^{i\theta}}{1 - (B/A)e^{i\theta}} = \frac{1 + e^{i\pi/2}/3}{1 - e^{i\pi/2}/3} = 0.80 + i\,(0.60)$$

- Impedance may depend on frequency – repeat measurements
  - Old graphical tool for matching impedance at ends = Smith Chart
    - Study the locus of a component's R and X versus frequency: see

https://www.antenna-theory.com/tutorial/smith/chart.php

- Can visualize locations of nodes with a Kundt Tube

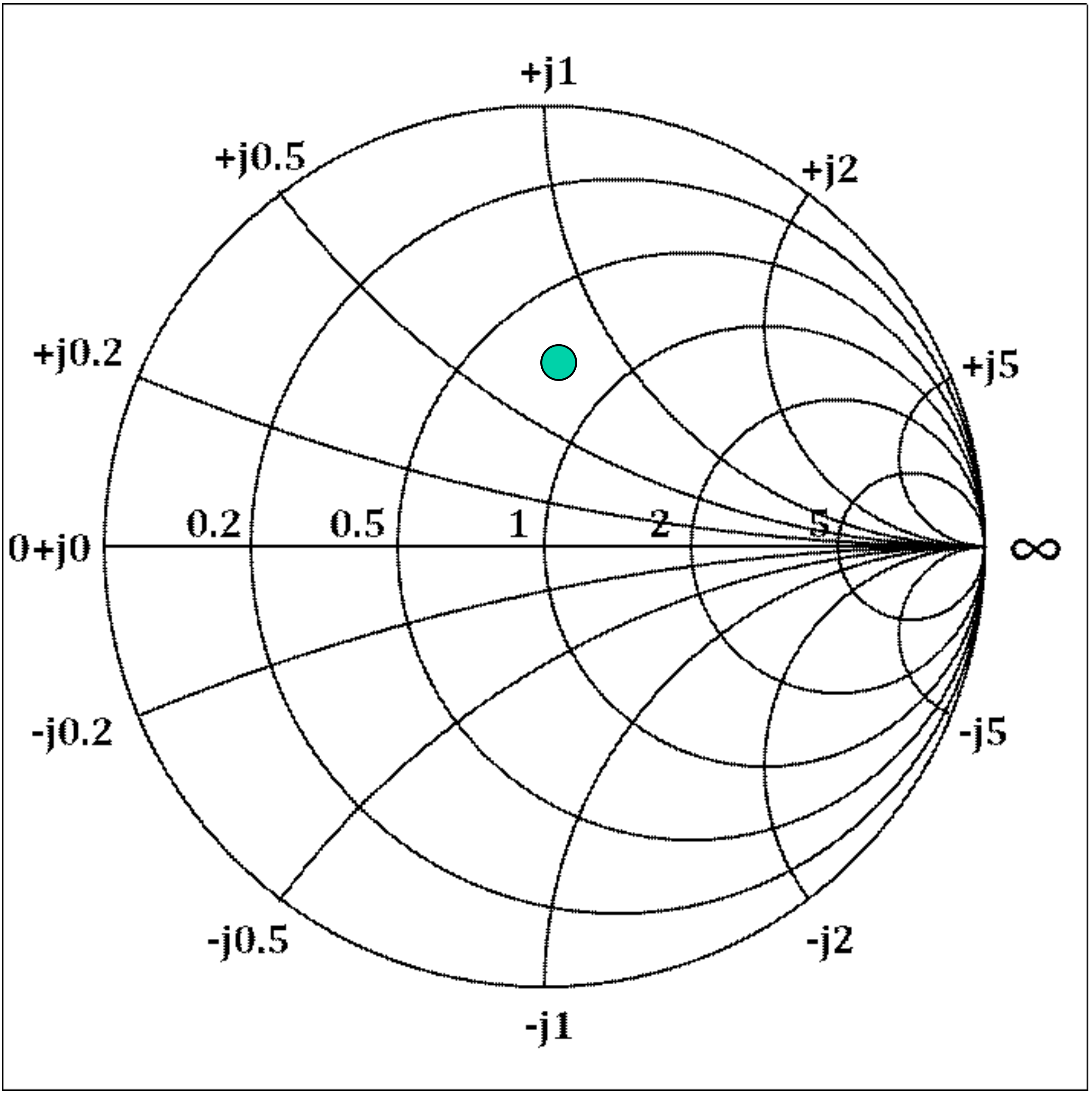
Smith chart for impedance matching

- Circles = lines of constant R
- Arcs = lines of constant X

- Example: Z = 0.8 + i 0.6

For R= constant, X varies with frequency:

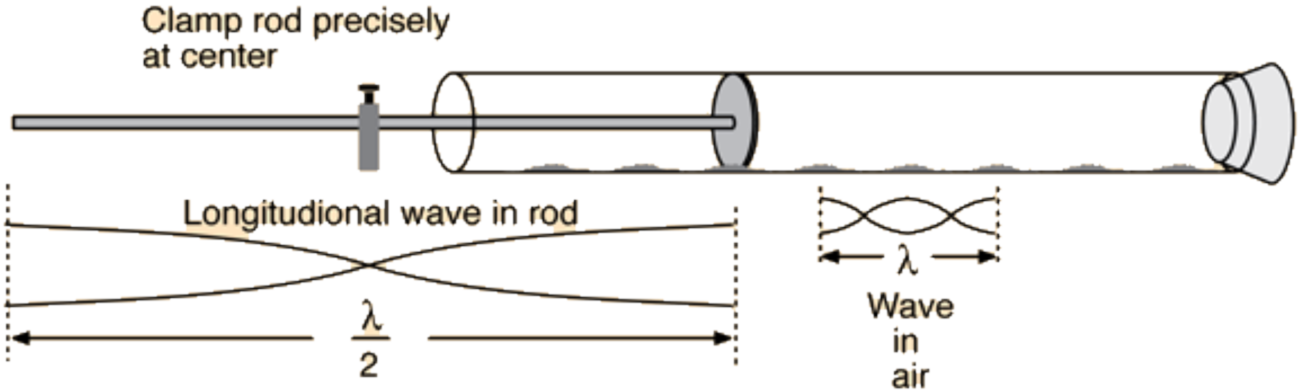
Find Z vs f by tracing circle for R=0.8



Kundt's tube

demo: https://www.youtube.com/watch?v=qUiB_zd9M0k

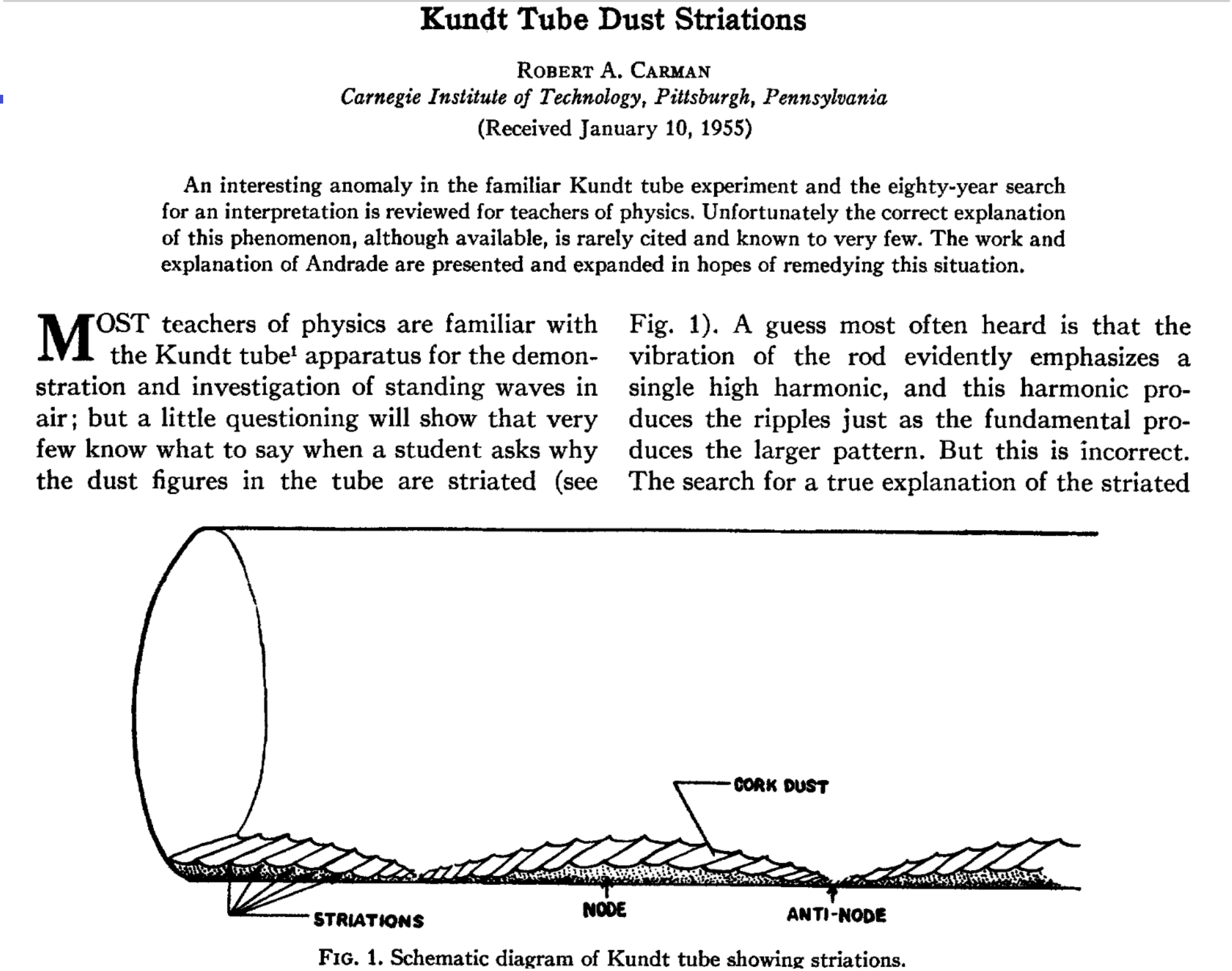
- Same idea as Chladni plates, applied to cylindrical tube
  - August Kundt (1866): measured speed of sound in gases and metals
  - Set up standing waves in glass cylinder filled with fine powder
    - Powder accumulates at nodes: node spacing = λ / 2
  - 19th C: set up wave of known λ by stroking metal rod on disk
    - Clamped at center → node → rod length = λ / 2 for longitudinal waves in metal rod



- Today: instead of passive end plug and stroked rod, use loudspeaker and signal generator

particle displacement $\xi(t) = 0$ at passive end, $\xi = a\cos(\omega t)$ at speaker

$$\xi(t) = A\sin\left(\frac{\omega}{c}x\right)\cos\left(\omega t\right) \text{ so nodes occur where } x = \frac{c}{2\pi f}n\pi, \quad n = 0,1,2...$$



Kundt Tube Dust Striations

Robert A. Carman
Carnegie Institute of Technology, Pittsburgh, Pennsylvania

(Received January 10, 1955)

An interesting anomaly in the familiar Kundt tube experiment and the eighty-year search for an interpretation is reviewed for teachers of physics. Unfortunately the correct explanation of this phenomenon, although available, is rarely cited and known to very few. The work and explanation of Andrade are presented and expanded in hopes of remedying this situation.

MOST teachers of physics are familiar with the Kundt tube[1] apparatus for the demonstration and investigation of standing waves in air; but a little questioning will show that very few know what to say when a student asks why the dust figures in the tube are striated (see Fig. 1). A guess most often heard is that the vibration of the rod evidently emphasizes a single high harmonic, and this harmonic produces the ripples just as the fundamental produces the larger pattern. But this is incorrect. The search for a true explanation of the striated

Fig. 1. Schematic diagram of Kundt tube showing striations.

Kundt’s tube puzzle: what causes the striations?

- K’s tube demo is performed everywhere, but few can answer “what are those closely spaced little columns of powder in the standing wave peaks?”!
  - Wrong answer: higher overtones in the tube (λ too small – need huge n)
- Answer: vortex formation due to failure of assumption that flow in tube is laminar
  - For full explanation, *see Kundt Tube Dust Striations*, Carman, Robert A., American journal of physics, 1955, Vol.23, p.505-507

Photographic studies by Andrade show that any particle suspended in the medium which does not share in the vibrational motion of the air column may become the center of a localized vortex motion (Fig. 2). Thus at proper conditions a pattern of such vortices will exist in the tube. A glance at

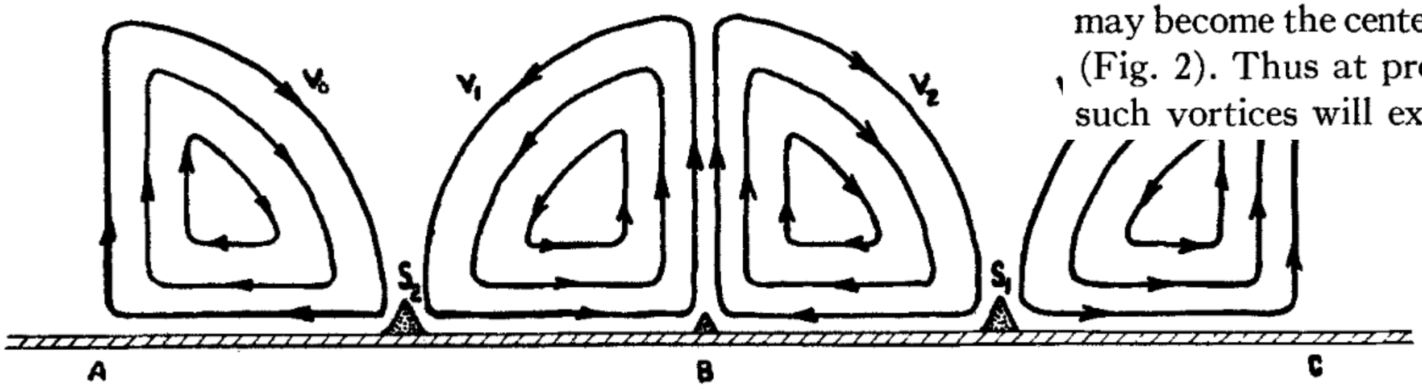
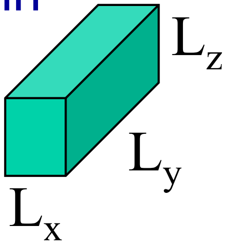


FIG. 3. Cross section of Kundt tube showing the ripple formation process. Dust is swept up at points A and B by vortices V_0 and V_1 to form a ripple at S_2. Similarly a ripple at S_1 is formed by vortices V_2 and V_3. Minor or secondary ripples are formed by the remaining dust at points A, B, and C.

Standing waves in rectangular cavities

- Sound boxes for musical instruments and building ducts act as cavity resonators for sound
  - We can apply similar procedure as with pipes, except now in Cartesian coordinates and with no openings
    - Cavity has dimensions L_x , L_y , L_z,
    - Assume walls are rigid (particle speed u=0 at wall)
    - Repeat procedure used for 2D case in membranes:



$$\nabla^2 p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \rightarrow \left.\frac{\partial p}{\partial x}\right|_{x=0,\,L_x} \left.\frac{\partial p}{\partial y}\right|_{y=0,\,L_y} = \left.\frac{\partial p}{\partial z}\right|_{z=0,\,L_z} = 0 \quad (u = 0 \text{ at walls})$$

  - Separation of variables:

$$p = X(x)Y(y)Z(z)\, e^{i\omega t} \rightarrow \left(\frac{d^2}{dx^2} + k_x^2\right)X = 0, \text{ same for } y,\ z$$

$$\text{separate constants must be related: } \omega / c = k^2 = k_x^2 + k_y^2 + k_z^2$$

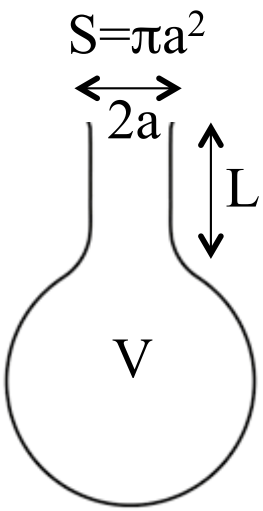
$$u = 0 \text{ at walls } \rightarrow p = A_{lmn}\cos\left(k_{xl}x\right)\cos\left(k_{ym}y\right)\cos\left(k_{zn}z\right)\exp\left(i\omega_{lmn}t\right)$$

$$\text{with } k_{xl} = l\pi / L_x,\ k_{ym} = m\pi / L_y,\ k_{zn} = n\pi / L_z, \quad \{l,m,n\} = 0,1,2...$$

$$\text{Allowed } \omega\text{s are quantized: } \omega_{lmn} = \sqrt{\left(l\pi / L_x\right)^2 + \left(m\pi / L_y\right)^2 + \left(n\pi / L_z\right)^2}$$

Helmholtz resonators

- Long−λ limit: if λ >> size of object, acoustic variables are ~ constant within it: "lumped acoustic element"
  - Spatial coordinates can be ignored in equations of motion
  - Object acts like a 1D harmonic oscillator
- Helmholtz resonator = simple lumped element
  - If λ >> L , then fluid in neck acts like lumped mass
    - $m = \rho_0 S L_{EFF}$ ($L_{EFF} \sim L + 1.5a$ if no flange)
  - If λ >>√S opening radiates like a simple source: resistance
    - $R_r = \rho_0 c k^2 S^2 / (4\pi)$ (see Kinsler for details)
  - If λ >> $V^{1/3}$ then acoustic *p* inside acts as stiffness element
    - $s = \rho_0 c^2 S^2 / V$ (see Kinsler for details)
  - Then



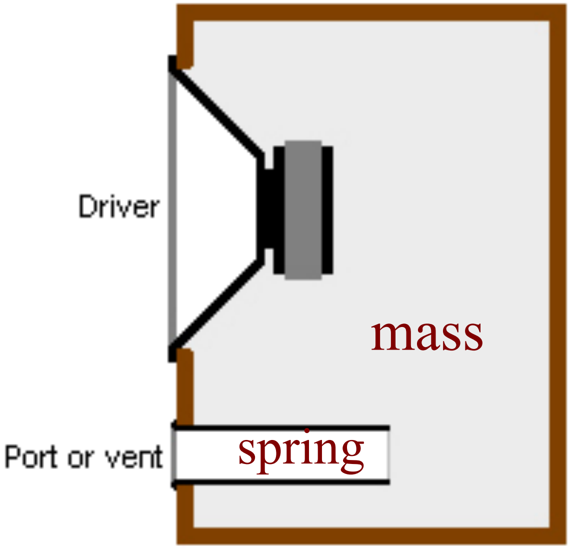
$$Z_m = R_r + i(\omega m - s/\omega) \rightarrow \text{resonant freq } \omega_0 = c\sqrt{\frac{S}{L_{EFF}V}}$$

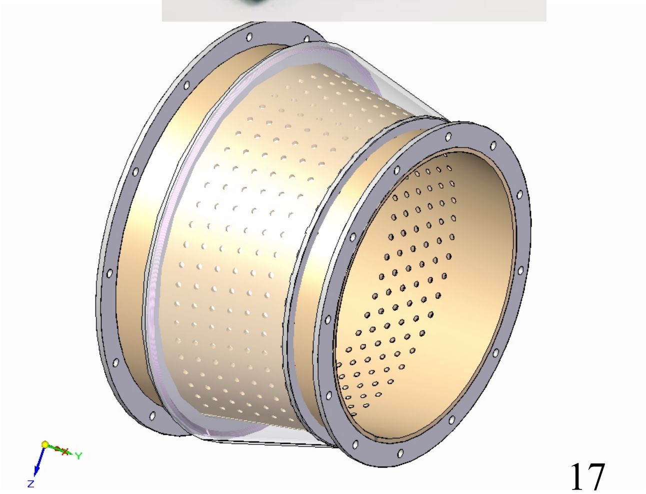
$$Q_{HELMHOLTZ} = \frac{\omega_0 m}{R_r} = 2\pi\sqrt{V\left(\frac{L_{EFF}}{S}\right)^3}$$

Here, we're using Q=quality factor (resonance sharpness) , not source strength!

Helmholtz resonator applications

- Loudspeaker cabinets
  - Bass reflex enclosure
    - Port lets the rear side of the speaker cone contribute; higher efficiency at low f's compared to a sealed box enclosure
- Motorcycle/car mufflers
  - Reduce noise, or "tune" tone
- Musical instruments
  - Ocarina is basically Helmholtz with selectable ports
- Aircraft engine noise reduction
  - Honeycomb liners reduce noise and drag
    - Compact array of resonators to absorb sound
- Mechanical bandpass filters





Resonant bubbles

- Bubbles in seawater absorb and scatter sound: another example of lumped-parameter acoustics
  - If radius of bubble *a* << λ, sound waves → radial oscillation

acoustic signal $p = -\rho_b c_b^2 \Delta V / V$ with ρ_b , c_b = interior gas properties

recall for gas, $\rho_b c_b^2 = \gamma_b P_0$, with P_0 = exterior pressure, $\gamma_b = C_P / C_V$

$\Delta V / V = -4\pi a^2 \xi / (4/3)\pi a^3$, and compressive force on surface is

$f = -p(4\pi a^2) \rightarrow f = -(4\pi a^2)\gamma_b P_0[-4\pi a^2 \xi / (4/3)\pi a^3]$, so

$f = -12\pi a \gamma_b P_0 \xi = -s\xi \rightarrow$ bubble's stiffness is $s = 12\pi a \gamma_b P_0$

  - Recall: for pulsating sphere in low frequency limit *ka* <<1, radiation impedance is

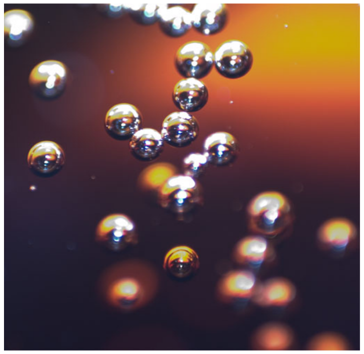
$Z_r = R_r + iX_r = 4\pi a^2 \rho c (ka)^2 + i4\pi a^2 \rho cka$; $ka << 1 \rightarrow R_r <<, X_r$

X_r acts like a mass: $m_r = X_r / \omega = 3\pi(4/3)a^3 \rho = 4\pi a^3 \rho$

resonance frequency $\omega_0 = \sqrt{\dfrac{s}{m_r}} = \sqrt{\dfrac{12\pi a \gamma_b P_0}{4\pi a^3 \rho}} = \dfrac{1}{a}\sqrt{\dfrac{3\gamma_b P_0}{\rho_0}}$

Resonant bubbles

- Heat transfer to water from bubble → non-adiabatic
  - "It can be shown" this acts like additional mechanical resistance:



At the resonant frequency, $\dfrac{R_m}{\omega_0 m_r} \approx 1.6x10^{-4}\sqrt{\omega_0}$

total impedance of resonant bubble $Z_m = (R_m + R_r) + i\left(\omega m - s/\omega\right)$

$$Q_{Bubble} = \frac{X}{R} = \frac{\omega_0 m}{R_m + R_r} = \frac{1}{k_0 a + 1.6x10^{-4}\sqrt{\omega_0}},$$

R_r = radiation impedance
R_m = mechanical impedance

Example: in seawater at 10 m depth, air bubble $a = 1\ mm$, $P_0 = 200kPA$, $c = 1500$ m/s, so

$$f_0 = \frac{\omega_0}{2\pi a} = \frac{1}{2\pi a}\sqrt{\frac{3\gamma_b P_0}{\rho_0}} = \frac{1}{6.3x10^{-3}}\sqrt{\frac{3(1.4)2x10^5}{1026}} = 4550\ Hz,$$

$$k_0 = \frac{2\pi}{\lambda} = \frac{2\pi c}{f_0} = \frac{6.3(1500)}{4550} = 2.07,\quad k_0 a = 0.002 << 1$$

$$Q = \frac{1}{k_0 a + 1.6x10^{-4}\sqrt{\omega_0}} = 34$$; power loss due to bubble (absorption + scattering)

$$\mathbf{P} = \frac{1}{2}U_0^2(R_m + R_r),\quad U_0 = \frac{F}{Z} = \frac{(4\pi a^2)|p|}{Z},$$ with p = sound wave amplitude

Acoustic impedance

- We've already encountered impedance in different contexts:
  1. Specific acoustic impedance z = p/u
     - Property of medium -- Useful for describing transmission of waves from one medium to another
  2. Radiation impedance Z_r = (force/speed)
     - $Z_r = zS$ -- Part of the mechanical impedance Z_m of vibrating system
     - Used for connecting radiation to vibrating source or load

Now: 3. Acoustic impedance Z=p/U (U = "volume velocity")

- Z= z/S -- Used for coupling radiation from vibrating surfaces into lumped elements, pipes, or horns

Lumped acoustic impedance:

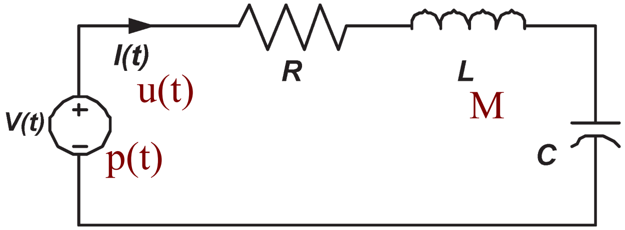
$$U = \frac{d\xi}{dt} S, \quad \xi = displacement, \quad S = surface\ area$$

$$Z = \frac{p}{U} \quad \text{units of Z are Pa} \cdot \text{s/m}^3, \quad (\text{"acoustic ohm"})$$

can write $Z = R + i\left(\omega M - \frac{1}{\omega C}\right)$, with $R = \frac{R_r}{S^2}$, M=$\frac{m}{S^2}$, C=$\frac{S^2}{s}$

Analogy to RLC circuit

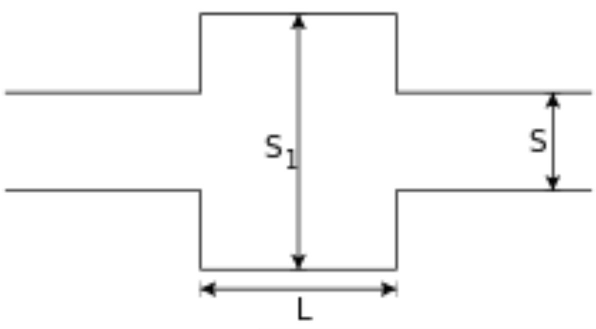
(L=mass, C=stiffness)



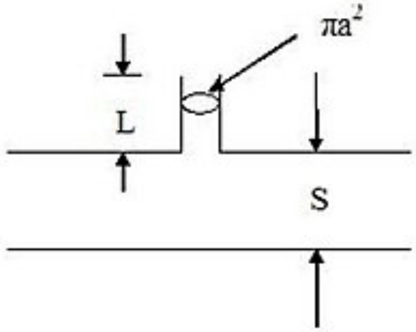
Physical acoustic filters

- Sound energy in a pipe can be diverted into wide parts of the pipe, or narrow parts, or an attached Helmholtz resonator :

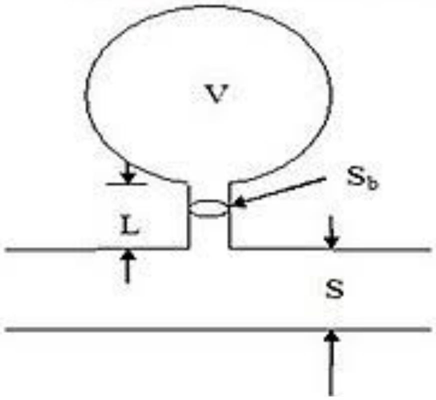
Low-Pass Filter Schematic



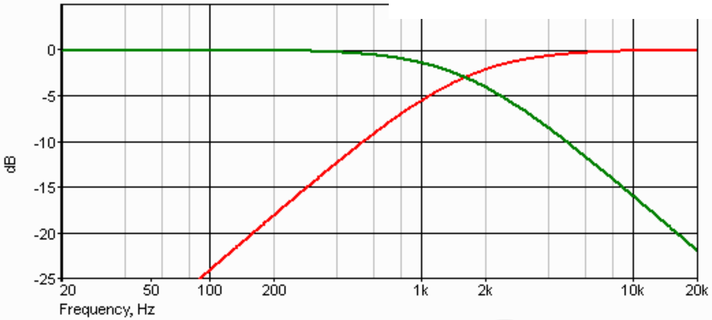
High-Pass Filter Schematic

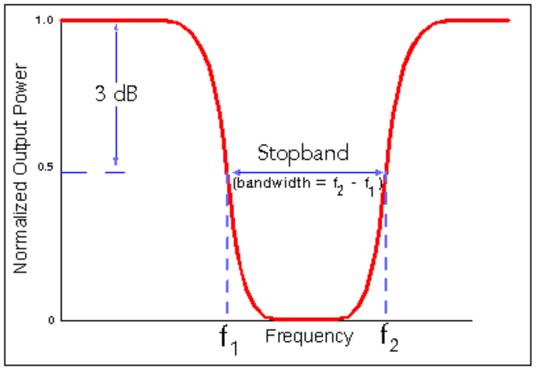


Band-Stop Filter Schematic

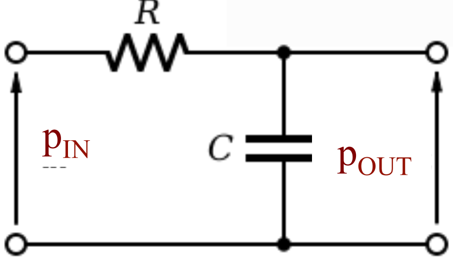


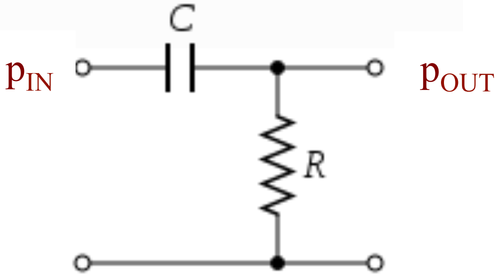
Electrical analogues:

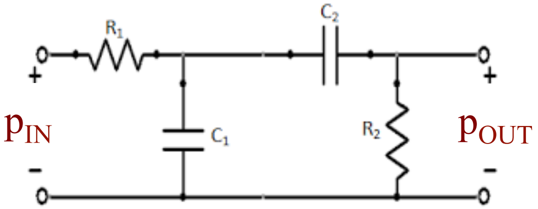




band-stop filter



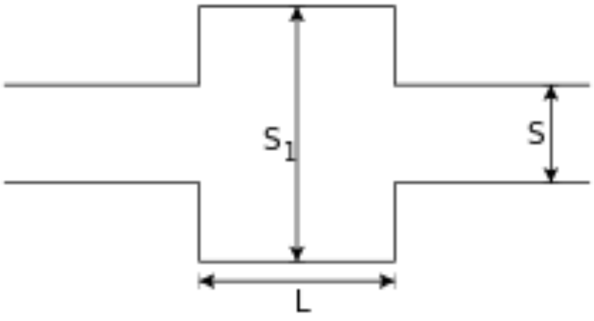




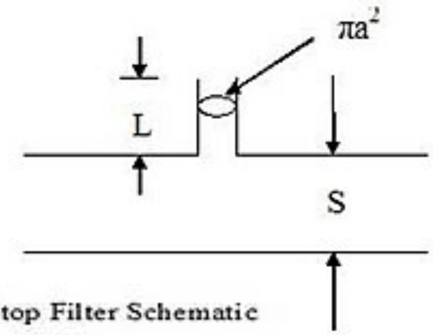
Physical acoustic filters

- Acoustic low-pass filter: insert an expansion chamber in the duct
  - simple model of a muffler
  - in architectural acoustics, plenum chamber in HVAC system
- Acoustic high-pass filter: insert a "T" junction, or a short side branch
  - Both the radius and the length of the side branch should be smaller than wavelength
- Acoustic band-pass filter: cavity attached to the side branch → Helmholtz resonator
  - Energy absorbed by resonator during one part of the acoustic cycle is later in the cycle.

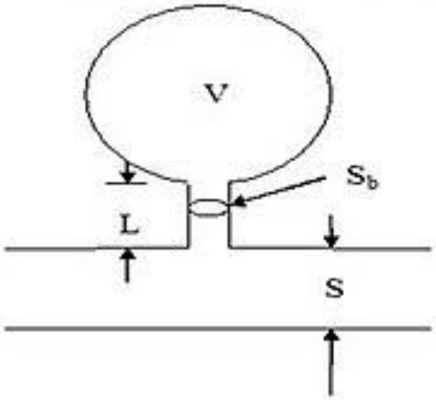
Low-Pass Filter Schematic



High-Pass Filter Schematic



Band-Stop Filter Schematic



Sound frequency; pitch and musical tones

- Frequency ranges
  - Audible – nominally 20 Hz to 20 kHz (actual range is closer to 50Hz-15kHz)
  - Infrasonic - below audible (below about 0.1 Hz we call it "vibration" !)
  - Ultrasonic - Above 20 kHz
- Speed of sound does not vary much with *f*
  - If v depended on f, sound signals would change significantly depending upon how far away you are
    - This is called "dispersion"
  - Small f dependence can be observed, for example in undersea sound transmission
    - A pulse with many frequencies in it will spread out in time as it travels
    - Pitch will vary – pulse becomes a "chirp"
- Perception of sound
  - Pitch = perceived frequency of sound
  - Associated with musical tones by our brain
  - JND = "just noticeable difference" in frequency ~0.4 Hz
  - Harmonic scales: eg in western music, "A above middle C" = 440 Hz, next A (one octave higher pitch) = 880 Hz - octave = doubling of base frequency)
  - "Equal temperament" scale: 12 tones per octave, each is 1.06 f of previous (factor = 12th root of 2)

Again: Interference = defining property of waves

- If you see interference effects, you are looking at waves !

  Case study: Isaac Newton thought light was a stream of particles

  Newton's "Opticks" (1687) explained all observations at the time

  Thomas Young (120 years later) observed interference effects with light

  Only waves could do that…

  Wave theory of light replaced Newton's particle theory

- Interference depends on phase relationship of overlapping waves
- Phase relationship depends on distance from source

  Recall: Phase at distance D from source = $2\pi\ (D/\lambda)$

  but sin/cos repeat every cycle, so all that matters is where we are relative to start of latest cycle: fraction of a cycle

  Phase at distance D from source = 2π [fractional part of (D/λ)]

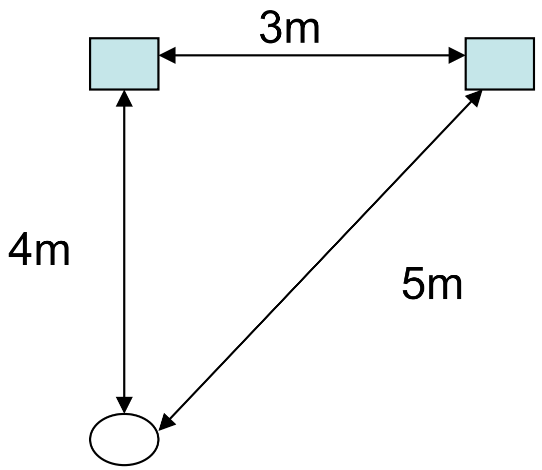
  Example: fractional part of $D/\lambda = 5.678 \rightarrow 0.678$ = mod(D/λ, 1)

Examples

- R and L channel loudspeakers, spaced 3m apart, are in phase: both speaker cones move forward or backward in sync together

Observer 4m away, parallel to L speaker hears constructive interference.

What f sound is being played?



$$D_1 = 4m \qquad D_2 = \sqrt{(3m)^2 + (4m)^2} = 5m$$

$$D_2 - D_1 = n\lambda$$

$$for \quad n = 1, \quad \lambda = D_2 - D_1 = 1m$$

$$f = \frac{c}{\lambda} = \frac{343\, m/s}{1m} = 343Hz$$

Any integer multiple of this f will also produce constructive interference at the observer’s location

Musical acoustics

- Musical terminology and scales
  - In addition to loudness, human perception of sound (other than simple monofrequency tones) is complex
  - Musical acoustics includes special terminology for factors such as
    - Pitch: perceived tone (not just frequency)
    - Timbre – tone quality
    - Temperament: definition of musical scales, relation to frequencies
  - Physics of human ears affects perception (more later)
  - Brain software creates "aural illusions" (more later)
    - Analogy to optical illusions caused by brain interpreting vision
  - Ohm's law of acoustics (1843): as interpreted by Helmholtz
    - All musical tones are periodic functions, but only sinusoidal vibrations are perceived as pure tones; other qualities are due to mixing (Fourier sum) of different sinusoids
    - Misleading: brain is not simply a Fourier analyzer

      → Musicians' distrust of physicists' analyses of music

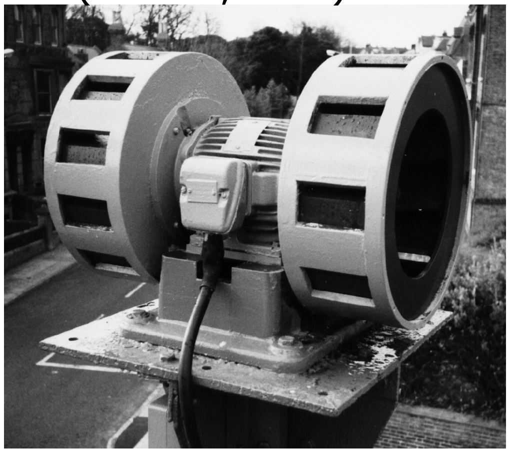
Georg Ohm

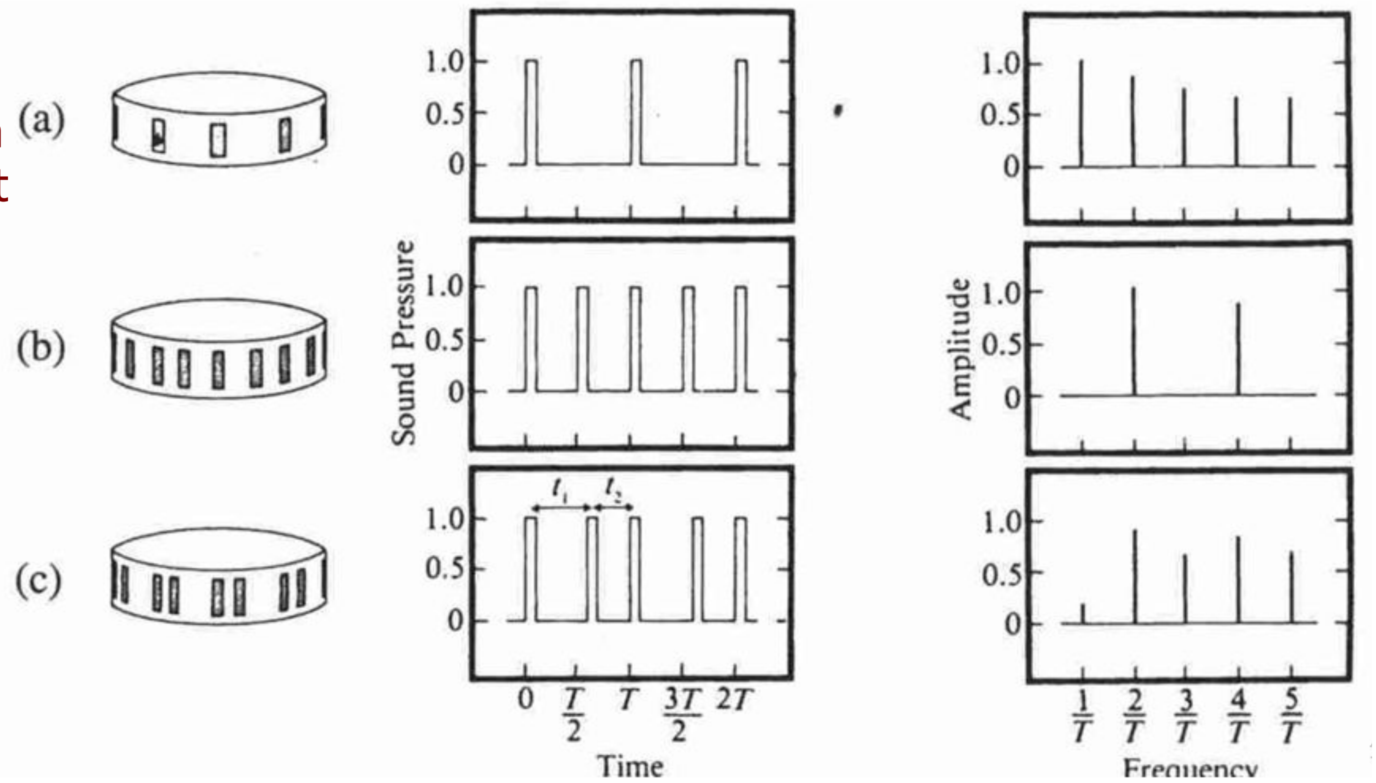
Pitch: not just frequency

- Pitch = characteristic of sound that determines position on scale
  - A. Seebeck's siren experiments (c. 1840)
    - Hole spacing + rotation speed → perceived pitch
      - Siren (b) sounds 1 octave higher than (a), but…
      - Siren (c) sounds ~ *the same* as (a)

Brain "fills in"
missing lower tones
because their effect on
Fourier spectrum is not
critical

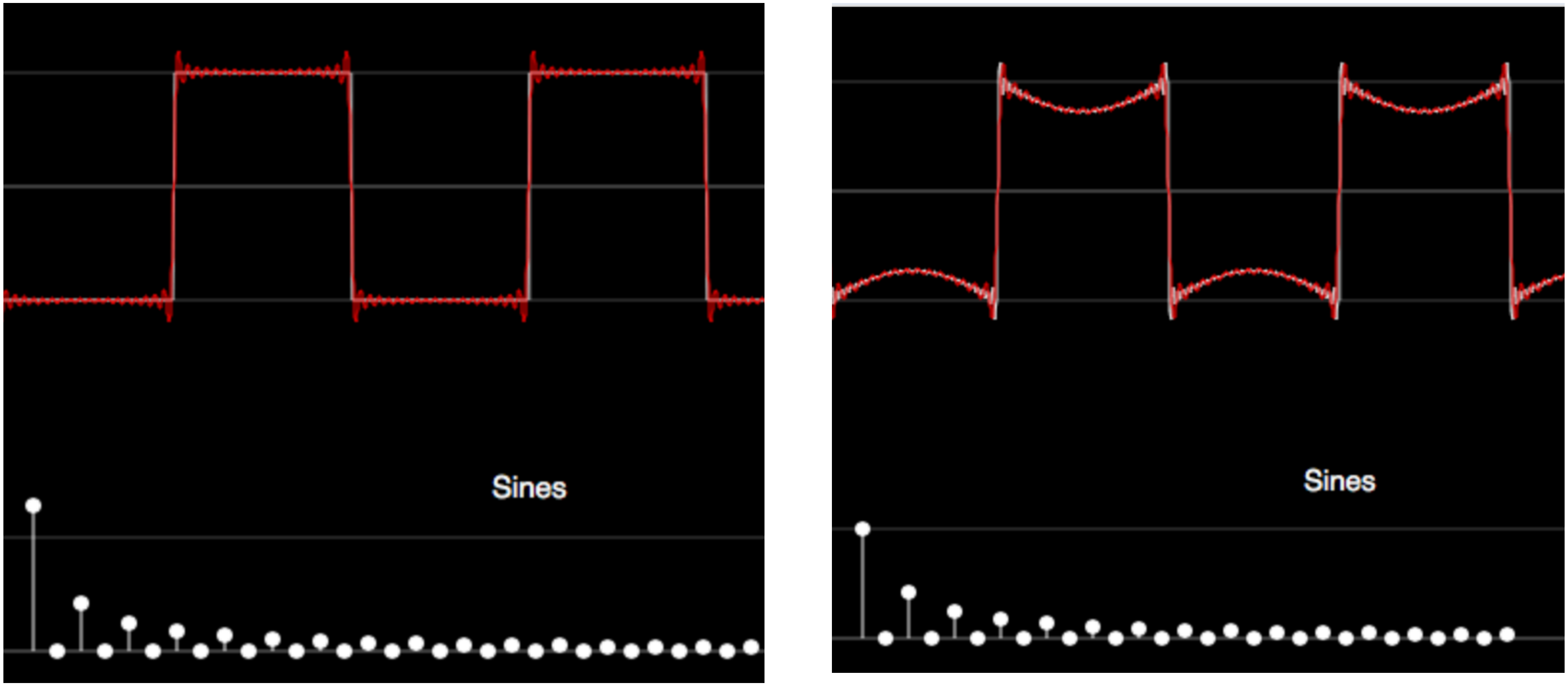
"Moaning Minnie"
(London, 1940)





Similarity of waveforms with low f’s missing

Signal + FT of a square pulse train, similar to Seebeck’s siren



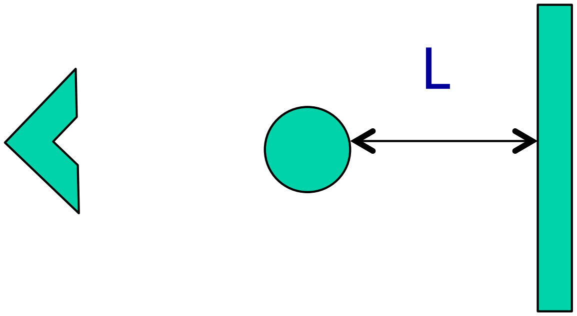
Full spectrum

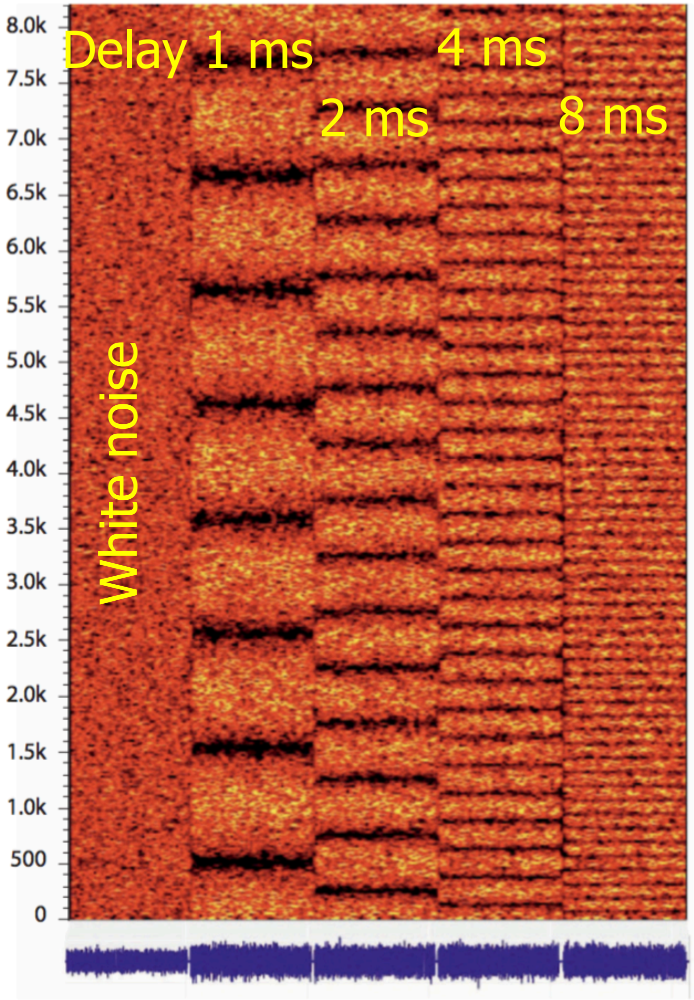
Fundamental missing

Another example: small loudspeaker in phone has poor response to actual 100 Hz pure tone (sinusoid) but creates perceived 100 Hz sound within complex signal

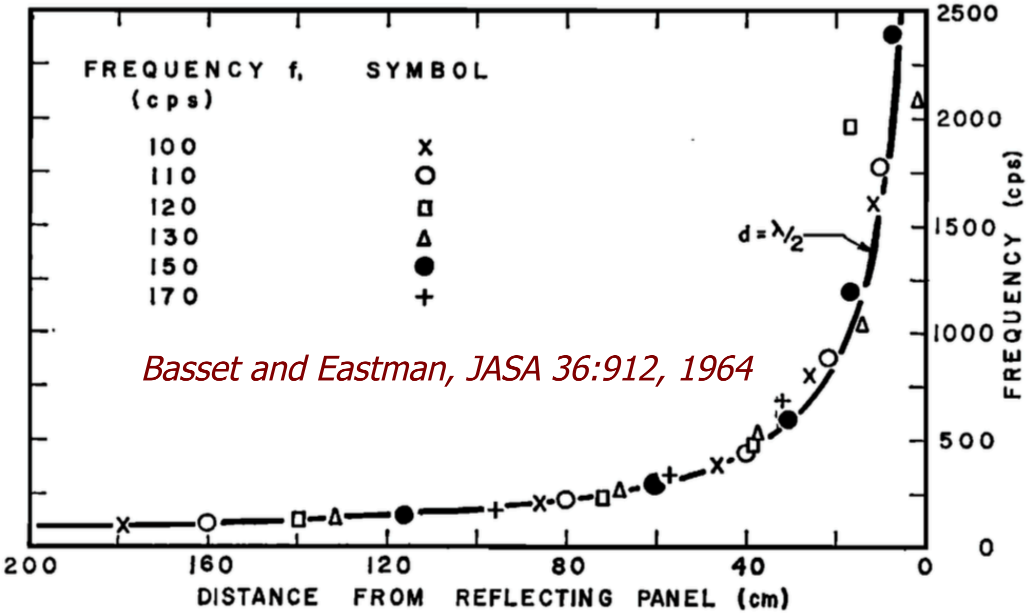
More on perceived pitch

- Interference → perceived tone
  - White noise reflected from wall
  - Perceived tone has f=1/T=c/L





Notice peaks at 1KHz, 500Hz, 250Hz, 125Hz intervals



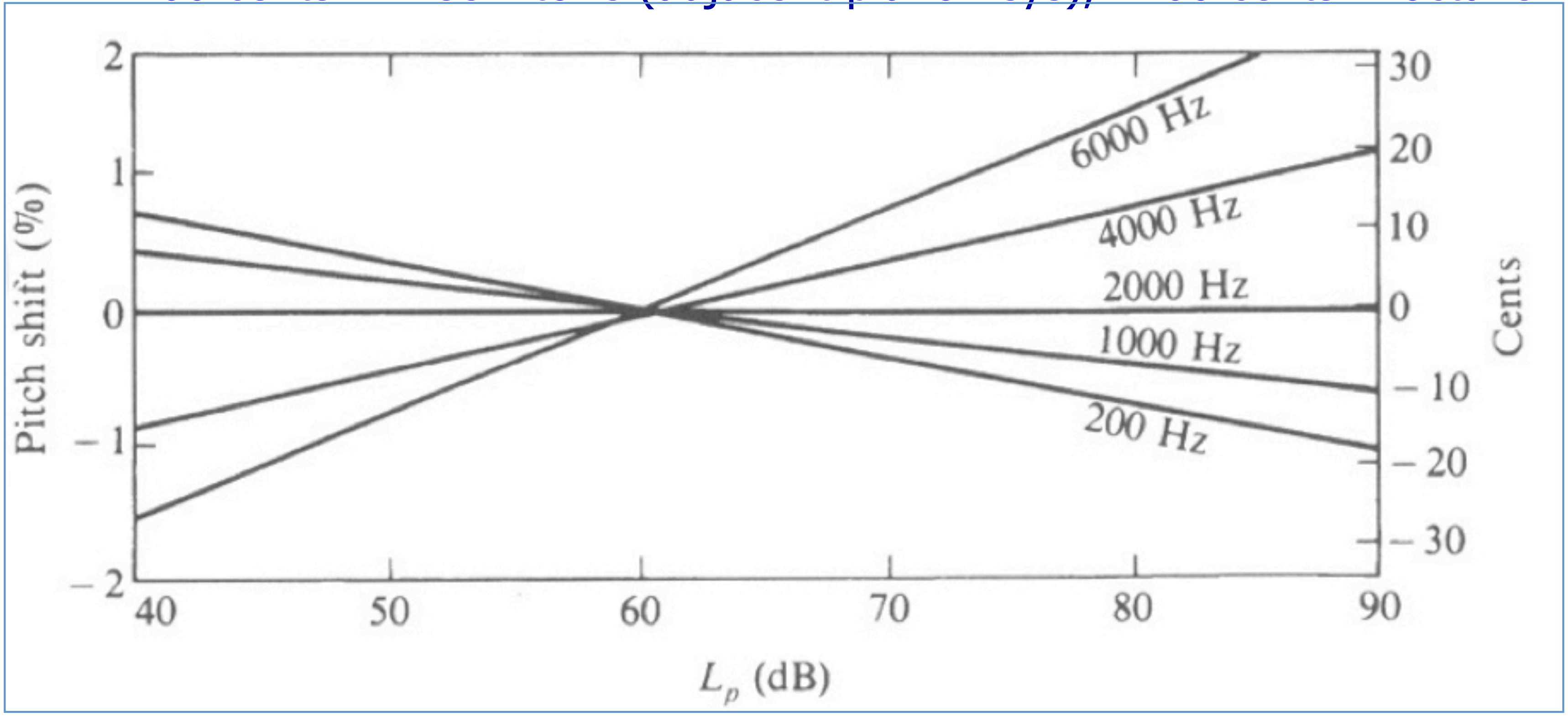
Basset and Eastman, JASA 36:912, 1964

FIG. 1. Variation of distance from reflecting panel with frequencies of comparison tones and their octaves.

More on perceived pitch

- Perceived pitch also depends on sound pressure level (SPL)
  - cent = logarithmic unit for musical intervals

    100 cents = 1 semitone (adjacent piano keys), 1200 cents = octave



For most people 3000 Hz will sound louder than 20 Hz sound with exactly the same amplitude.

Hearing is optimized for sounds in the 2000--4000 Hz range. Frequencies in this range seem louder than other sounds with equal amplitude.

Shepard tone illusion

https://www.youtube.com/watch?v=BzNzgsAE4F0

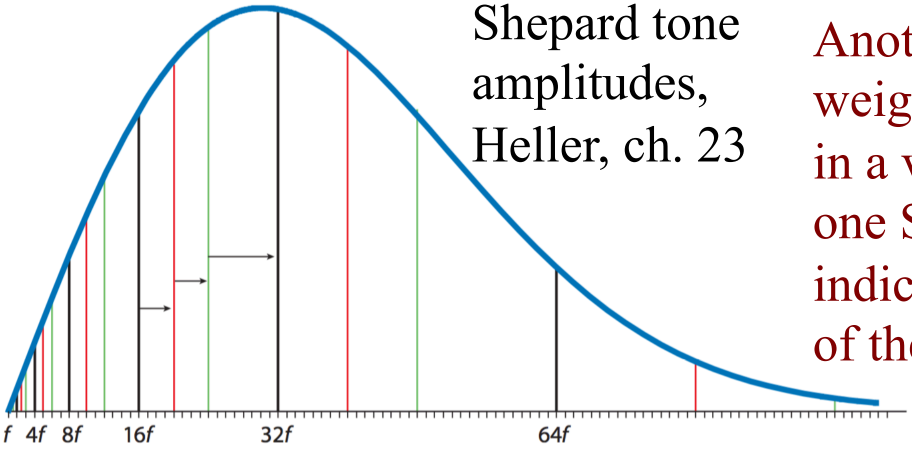
- Shepard tones are heard as pitch rising continually
  - After 12 semitones=1 octave, perceived pitch returns to beginning
    - Normal scale ("equal-tempered", more later) rises by factor $2^{1/12}$ for each semitone, each note with full set of partials at $f_n = n f_1$
    - Shepard includes only those that are powers of 2 times f_1 ,

      $f_n = 2^n f_1$, $n = 0,1,2,...$

      So frequencies in 1st run up the scale are

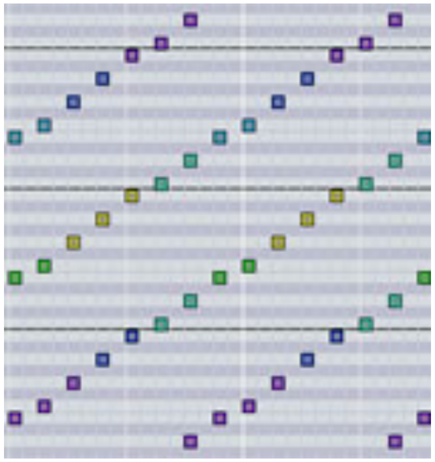
      $f_{m,n} = 2^{m/12} 2^n f_0$; $m = 0,1,...11$ (note in sequence);

      $n = 0,1,2,....$ (repetition of sequence = octave)
  - Modulating the amplitudes of successive octaves → repeated perceived pitches: for m=0: f_0, $2f_0$, $4f_0$, $8f_0$,…, when m=12, repeat the same set!



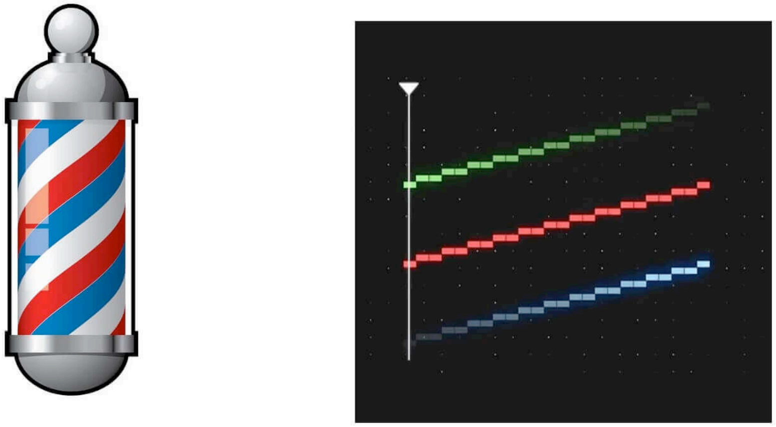
Shepard tone amplitudes, Heller, ch. 23

Another way to display weights: Each set of squares in a vertical line composes one Shepard tone. Color indicates increasing loudness of the note, purple to green.



Shepard illusion.

- Called “The Sonic Barber Pole” for the visual version of a seemingly constantly rising illusion.

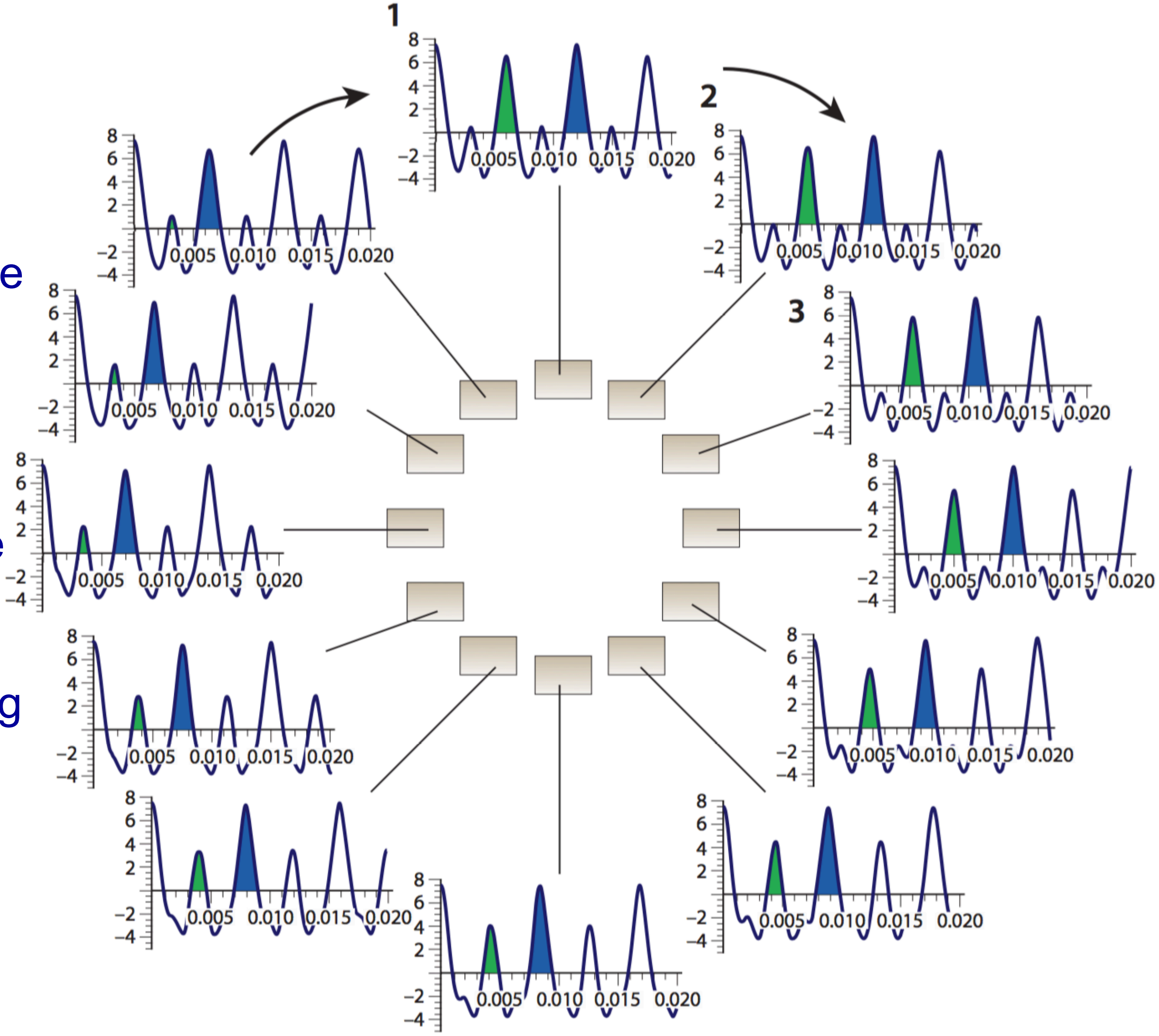


- Works both ways: also falling Shepard tone

https://www.youtube.com/watch?v=u9VMfdG873E

Shepard tone illusion diagram

- Each clockwise step is a semitone higher in pitch, shifts the autocorrelation peaks left, with small changes in their shape.
- They appear earlier in time and correspond to higher pitch.
- When the first two tall peaks are about equal in height, they are an octave apart, but the peak closer to *t* = 0 starts to diminish in height, gradually making the lower pitch more dominant.
- New peaks arrive after 12 steps, to exactly reproduce the first autocorrelation function.



Pitch standards for music

- Today: A440 or A4 (A above middle C), with f = 440 Hz is the general tuning standard for musical pitch
- Not so until 20th century! Pitch standard was subject of bitter fights…
  - Handel’s tuning fork was 422.5 Hz
  - 1859 French government commission (Berlioz, Rossini et al) chose 435 Hz
  - Verdi wanted to stop “creeping pitch” rise, suggested 432 Hz, based on…
  - “Scientific pitch” definition had all C’s powers of 2 (128 Hz, 256, 512, etc)
    - So A4 ~ 431 Hz

Lyndon LaRouche, leader of “socialist worker party” cult (died last week), had his followers lobby for “Verdi tuning” and proposed a law in Italy “to impose scientific notation on state-sponsored musicians that included provisions for fines and confiscation of all other tuning forks.” (Wikipedia, *Scientific pitch*)

Note	**f, (Hz)**
C0	16
C1	32
C2	64
C3	128
C4	**256**
C5	512
C6	1024
C7	2048
C8	4096
C9	8192