PHYS 536

R. J. Wilkes

Session 10

Musical acoustics: timbre, beats, scales, temperament

Musical instruments: Wind instruments,

String instruments

2/02/2023

Course syllabus and schedule – updated

See : http://courses.washington.edu/phys536/syllabus.htm

Session	date	Day	Readings:	K=Kinsler, H=Heller	Topic
8	26-Jan	Thu	K: Ch. 7	H: Ch. 7	Absorption losses; Pulsating spheres and simple sources; pistons and dipoles; Near field, far field; Radiation impedance; Waves in pipes ***UPDATED BELOW HERE:***
9	31-Jan	Tue	K. Ch. 8-10	H: Ch. 13	Rectangular cavities; Helmholtz resonators; Resonant bubbles; Acoustic impedance; physical acoustic filters; Doppler effect; Interference effects
10	2-Feb	Thu	K. Ch 9	H: Chs. 23-25	Musical acoustics: pitch, musical tones and frequency; timbre; beats
11	7-Feb	Tue		H: Chs. 16, 18	Musical instruments: winds and string instruments
12	9-Feb	Thu		H: Chs. 17, 19	Musical instruments: piano, human voice REPORT 1 PAPER DUE by 7 PM; REPORT 2 PROPOSED TOPIC DUE
13	14-Feb	Tue	K. Ch. 11	H: Ch. 21	Human hearing: the inner ear; pitch perception; acoustics of speech
14	16-Feb	Thu	K. Ch. 12	H: Chs. 21-22	Decibels and sound level measurements Environmental acoustics and noise criteria; industrial and community noise regulations; noise mitigation;
15	21-Feb	Tue	K. Chs. 13-14	H: Chs. 27-28; Ch. 6	Room acoustics; Transducers for use in air and water: Microphones and loudspeakers; hydrophones and pingers; Underwater acoustics: sound absorption underwater, the sonar equation
16	23-Feb	Thu	K. Ch 15		Underwater acoustics applications: acoustical positioning, seafloor imaging, sub-bottom profiling; Course wrap-up: review
17	28-Feb	Tue			Student report 2 presentations
18	2-Mar	Thu			Student report 2 presentations
19	7-Mar	Tue			Student report 2 presentations
20	9-Mar	Thu			Student report 2 presentations. TAKE-HOME FINAL EXAM ISSUED
--	17-Mar	Fri			FINAL EXAM ANSWERS DUE by 5 PM

Tonight

Class is over after you turn in your take-home exam. No in-person final exam during finals week.

Announcements

- Schedule of topics has been rearranged – pls check readings
- Due Thursday **next week**!
  - Papers for project 1
  - Proposals for project 2 presentation

From last time

Pitch standards for music

- Today: A440 or A4 (A above middle C), with f = 440 Hz is the general tuning standard for musical pitch
- Not so until 20th century! Pitch standard was subject of bitter fights…
  - Handel's tuning fork was 422.5 Hz
  - 1859 French government commission (Berlioz, Rossini et al) chose 435 Hz
  - Verdi wanted to stop "creeping pitch" rise, suggested 432 Hz, based on…
  - "Scientific pitch" definition had all C's powers of 2 (128 Hz, 256, 512, etc)
    - So A4 ~ 431 Hz

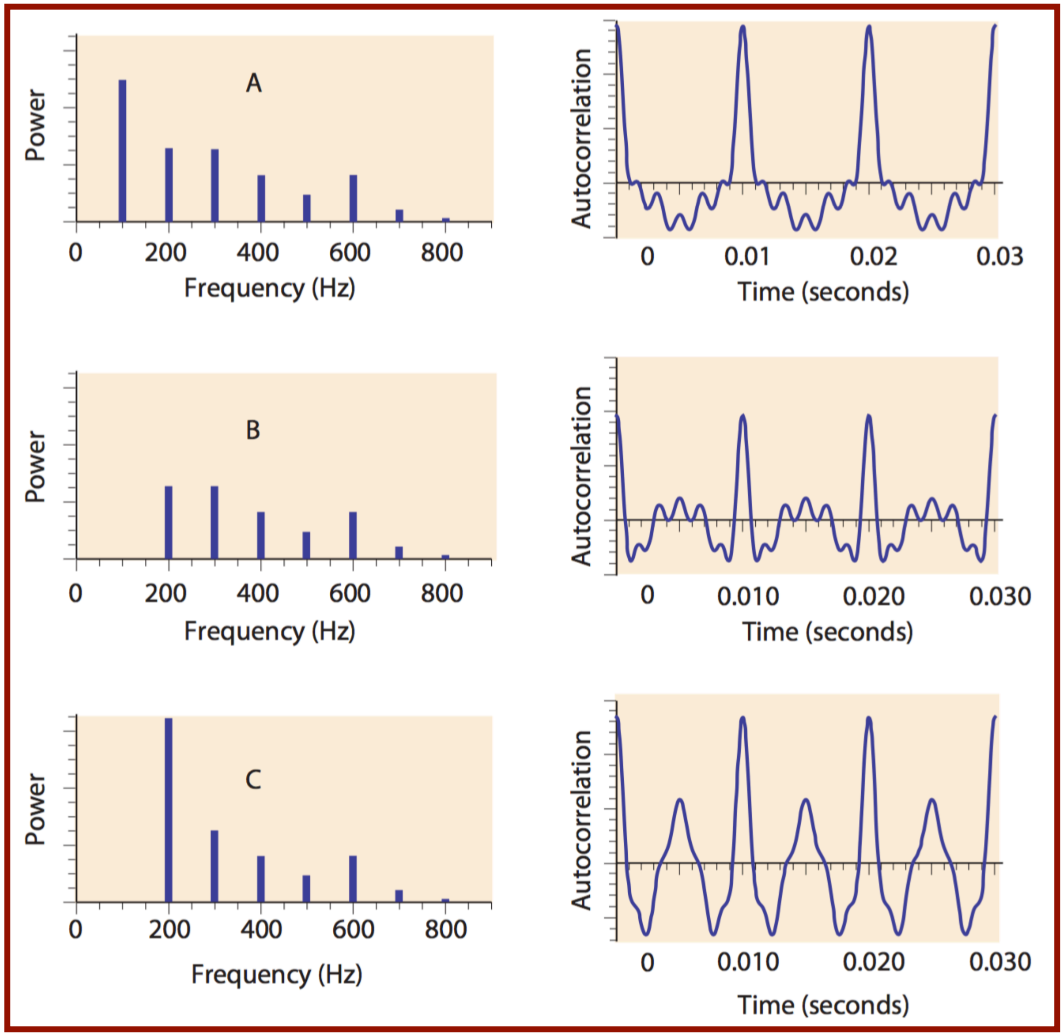
Lyndon LaRouche, leader of "socialist worker party" cult (died last week), had his followers lobby for "Verdi tuning" and proposed a law in Italy "to impose scientific notation on state-sponsored musicians that included provisions for fines and confiscation of all other tuning forks." (Wikipedia, *Scientific pitch*)

Note	f, (Hz)
C0	16
C1	32
C2	64
C3	128
C4	**256**
C5	512
C6	1024
C7	2048
C8	4096
C9	8192

Pitch and autocorrelation

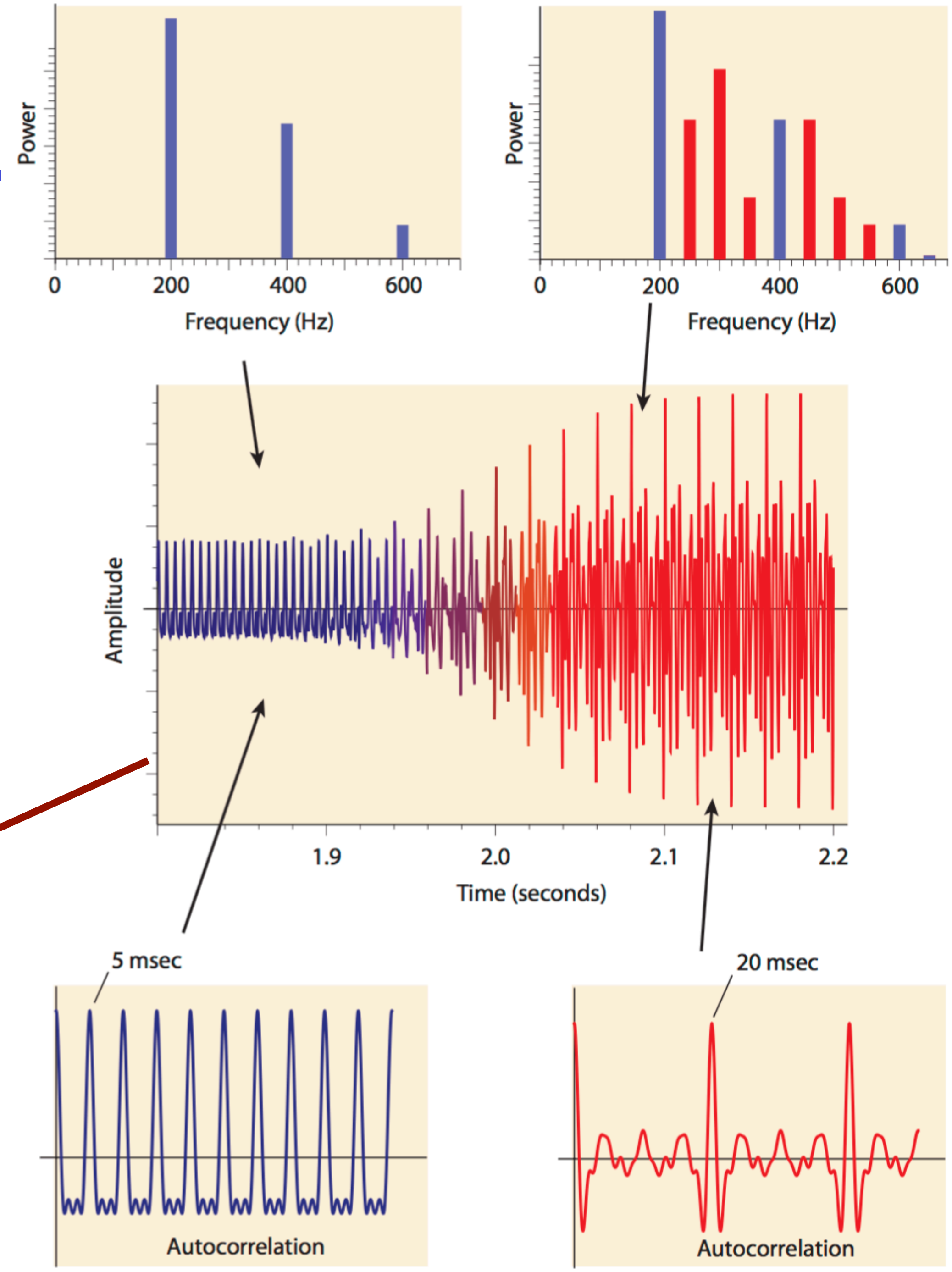
- Seebeck's sirens show perceived pitch may be frequency of missing fundamental component
  - "Missing fundamental" effect – Heller calls it "residue pitch"
- Signal with period T has autocorrelation peaks at t=nT, n=0, 1, 2,…
  - Same true for $(\text{signal})^2$

A. Power spectrum of sound with f_1 =100 Hz and several partials
B. Fundamental removed → same autocorrelation peaks
C. Increase power in f_2 =100 Hz → autocorrelation peaks appear at half-intervals → perceived as sound with 200 Hz fundamental



Pitch and autocorrelation

- Example in Heller textbook (p. 448)
- Initial signal has 3 partials*, 200/400/600 Hz
  - Perceived as 200 Hz sound
- Second signal has additional partials at 50 Hz intervals
  - Perceived as 50 Hz tone



- Partials = overtones

Not necessarily harmonics!

Recall: driven oscillator settles down to driver f, but oscillator mechanics affect overtone frequencies – "color"

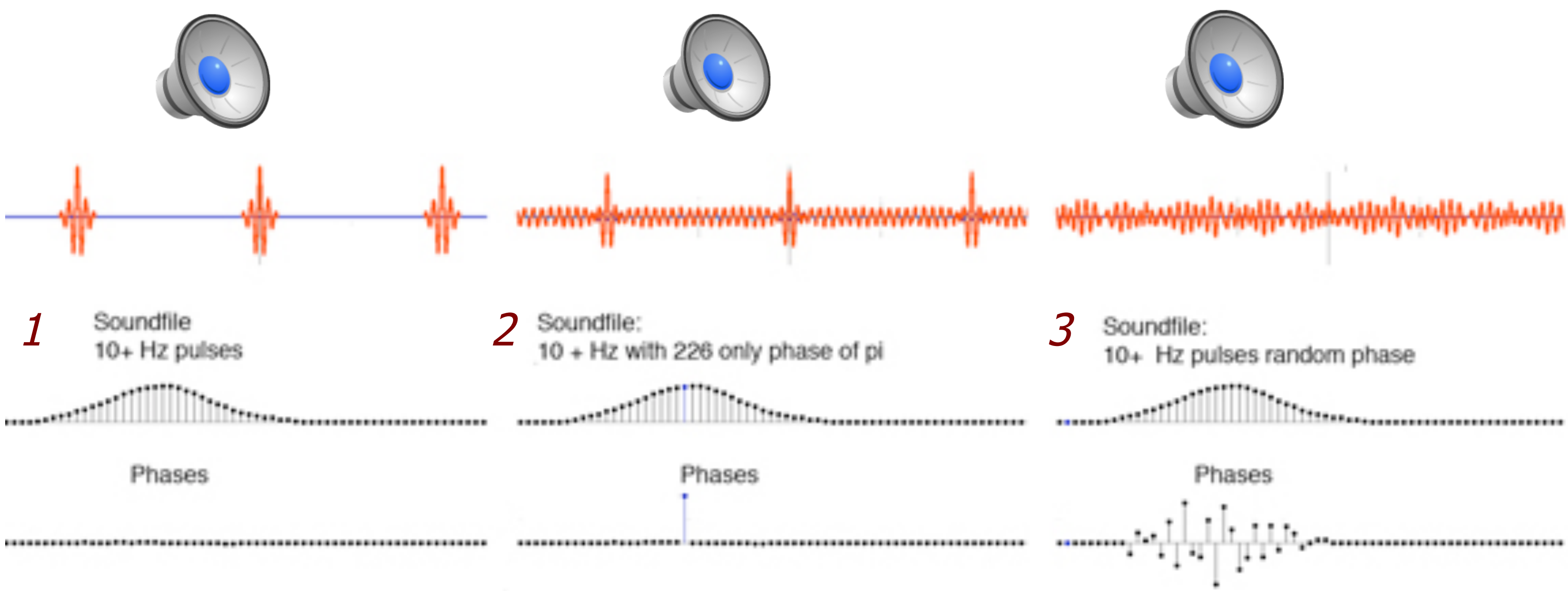
“Pitch” of subsonics

- From Heller book:

Extend the definition of pitch into the "counting" realm below 20 Hz:

1. Short pulses at 0.1 sec intervals – sharp clicks have broad spectrum
2. Same sound, but add phase shift of one component (200 Hz)
3. Same, but now add random phase shifts to many components

All three sound files have the same autocorrelation function, looking much like the first sound trace, starting at a peak of one of the pulses.



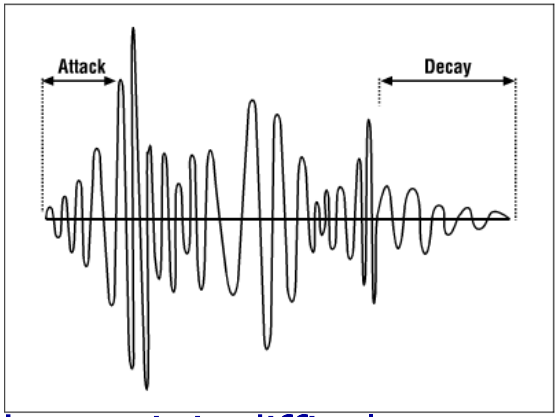
Timbre – tone quality

- Timbre – tone quality or "color"
  - Criterion: "attribute that allows listener to judge two sounds dissimilar by criteria other than pitch, loudness and duration"
  - Depends primarily on spectrum but also on waveform, SPL, frequency range, and envelope shape
    - Descriptor scales used for timbre :

      Dull ←→ Brilliant

      Cold ←→ Warm

      Pure ←→ Rich



  - Attack = onset of the sound (eg, bow on string)
    - If attack is deleted leaving only the sustained tone, it is difficult to identify the instrument
    - Attack-decay envelope = shape of amplitude envelope of sound
  - Beating: modulations of amplitude due to summation of partials with similar f's

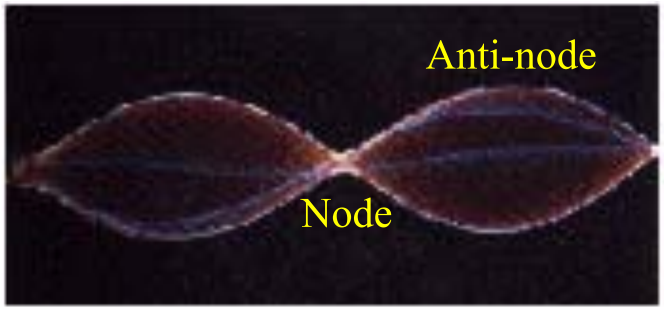
Timbre

From Heller book

- *Timber is unlike pitch or loudness in that there is no one-dimensional scale (like frequency for example) that it can be mapped onto.*
- *It has been conjectured that timbre is 37 dimensional (color perception is three dimensional: the amount of red, blue, an green). 37 is the number of independent critical bandwidths on the basilar membrane.*
  - *There are excitations of 37 separate regions on the basilar membrane* [we will discuss the ear's structure later]*, and timbre would be determined by this pattern of loudness in these 37 regions. In fact only perhaps the lowest 15 or 20 regions play a large role in pitch; extremely high frequencies are less important.*
- *Other things that matter:*
  - *Is the sound periodic?*
  - *Does the envelope of the sound fluctuate, or is it constant?*
  - *What the preceding sounds are like? **
  - *Is the sound ramping up or down in loudness (another envelope issue)?***

** Segment of sound of bell* ***Rhapsody in Blue opening played forward, then backward*

Beats



- Interference pattern like this shows locations of nodes and antinodes in space.
- We can also create a moving interference pattern, so a stationary observer hears cyclic intensity changes as maxima pass:

  This is called beating, or a beat frequency

- Beats are heard if waves with similar frequencies overlap at the observer's location, x: then at that spot, amplitude vs time is

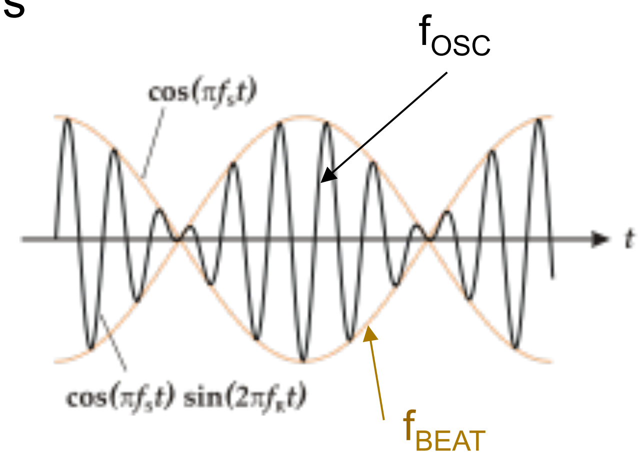
$$y_1(t) = A\cos(2\pi f_1 t), \quad y_2(t) = A\cos(2\pi f_2 t)$$

$$y_1 + y_2 = A\cos(2\pi f_1 t) + A\cos(2\pi f_2 t)$$

Trig fact: $\cos(a) + \cos(b) = 2\cos\left(\frac{a-b}{2}\right)\cos\left(\frac{a+b}{2}\right)$

$$y_1 + y_2 = 2A\cos\left(2\pi\left[\frac{f_1 - f_2}{2}\right]t\right)\cos\left(2\pi\left[\frac{f_1 + f_2}{2}\right]t\right)$$

$$f_{BEAT} = |f_1 - f_2| \qquad f_{osc} = \frac{f_1 + f_2}{2}$$



The sum has a base frequency f_{OSC}, modulated by an envelope of frequency f_{BEAT}

Notice: f_{BEAT} is twice the f in the cosine function: Envelope goes from max to min in ½ cycle, so frequency of pulsation is $2\left[\frac{f_1 - f_2}{2}\right] = f_1 - f_2$

Beats

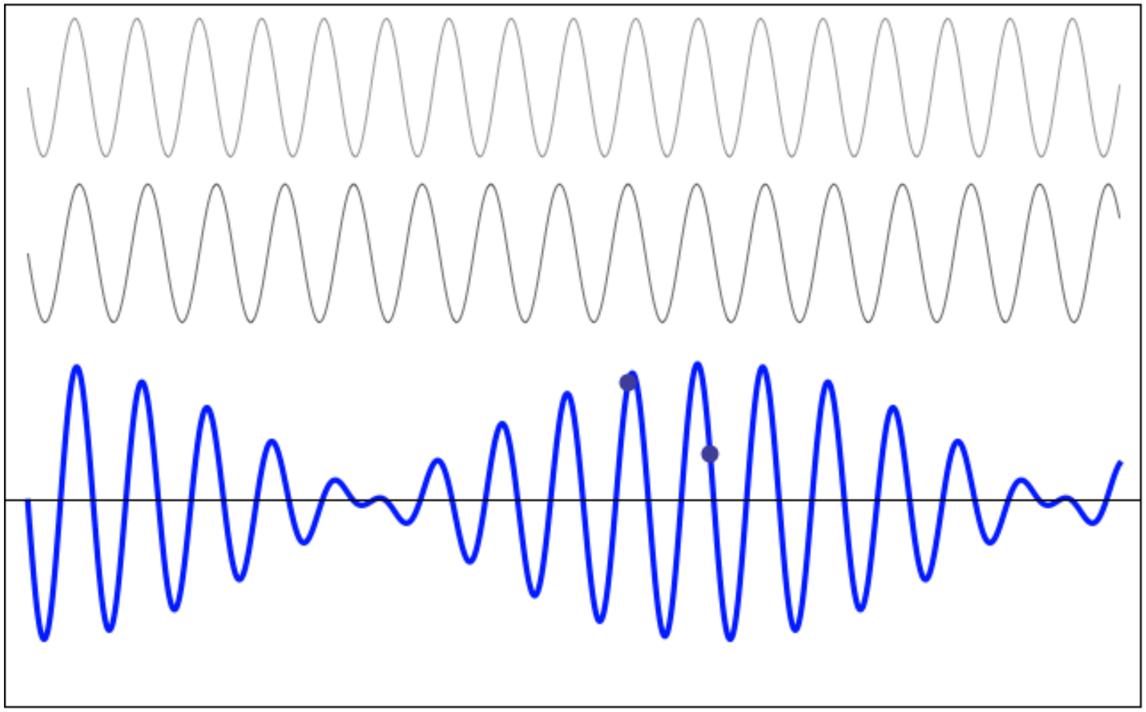
- Below: 2 waves with slightly different f's are travelling to the right.
- The waves are in the same medium, so have the same speed.
- Superposition sum wave has the same direction and speed as the two component waves, but its local amplitude depends on their relative phase.

Beat wave oscillates with the average frequency, and its amplitude envelope varies with the difference frequency.

The dots show how y vs t varies at two fixed x positions

$$f_{BEAT} = |f_1 - f_2|$$

$$f_{OSC} = \frac{f_1 + f_2}{2}$$

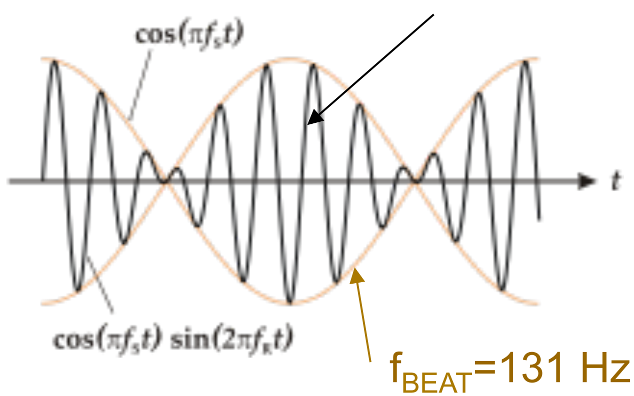


Beats

- Example: pluck 2 notes on a guitar: middle C (261 Hz) and G (392 Hz)

$$f_{BEAT} = |f_1 - f_2| = (392\text{ Hz} - 261\text{ Hz}) = 131\text{ Hz}$$

$$f_{OSC} = \frac{f_1 + f_2}{2} = 326.5\text{ Hz}$$



Notes	Frequency (octaves)				
A	55.00	110.00	220.00	440.00	880.00
A#	58.27	116.54	233.08	466.16	932.32
B	61.74	123.48	246.96	493.92	987.84
C	65.41	130.82	261.64	523.28	1046.56
C#	69.30	138.60	277.20	554.40	1108.80
D	73.42	146.84	293.68	587.36	1174.72
D#	77.78	155.56	311.12	622.24	1244.48
E	82.41	164.82	329.64	659.28	1318.56
F	87.31	174.62	349.24	698.48	1396.96
F#	92.50	185.00	370.00	740.00	1480.00
G	98.00	196.00	392.00	784.00	1568.00
A♭	103.83	207.66	415.32	830.64	1661.28

base frequency modulated by an envelope:

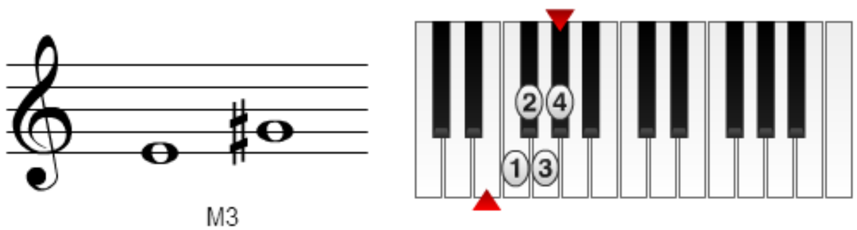
f_{OSC}=327Hz = ~ E in next octave

f_{BEAT}=131 Hz = C an octave below middle C

Musical scales and temperament

- "Just* intonation": tuning system that involves intervals limited to integer ratios of fundamental f – examples: ** As in "justice""*
  - "5-limit tuning": notes multiply f_1 (the base note) by products of integer powers of 2, 3, or 5
    - Powers of 2 = octaves (f ratio 2:1), powers of 3 = intervals of perfect fifths (3:2), powers of 5 = intervals of major thirds (5:4)
  - Pythagorean scale: only pure octaves and perfect fifths allowed

major third



- If only integer ratios are used, chords sound in tune only if based on the same fundamental frequency
  - changing musical keys is not possible.

Frequencies vs temperament

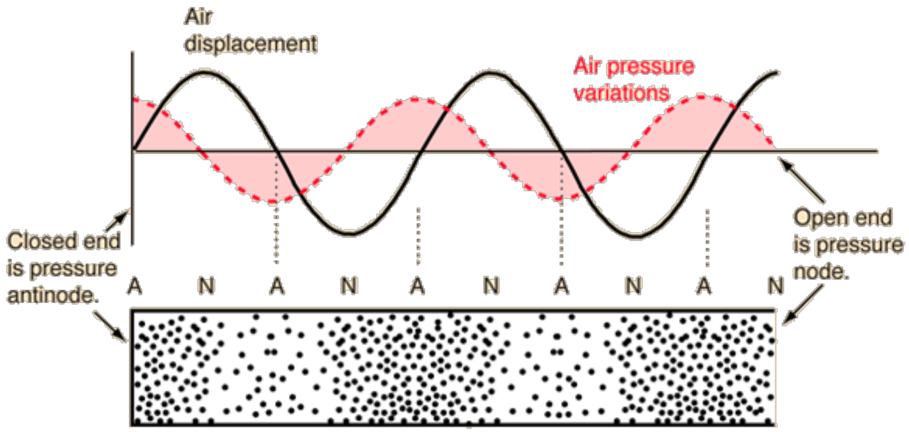
- Temperaments allow fixed-pitch instruments (eg, piano, harpsichord) to be played in different musical keys without dissonance
  - "Well-tempered" (eg Bach) = some keys are more in tune than others, but all can be used
  - Equal-tempered (modern standard) = every pair of adjacent notes has the same ratio of f's: $2^{1/12}$ → pitch is perceived as ~ log(f)

Pythagorean		**Just Major**		**Equal-tempered**	
Note	*Hz*	*Note*	*Hz*	*Note*	*Hz*
C	523.25	C	523.25	C	523.25
B	496.67	B	490.55	B	493.88
B^b	465.12	B^b	470.93	$B^b/A^\#$	466.16
A	441.49	A	436.05	A	440.00
A^b	413.42	A^b	418.60	$A^b/G^\#$	415.30
G	392.44	G	392.44	G	392.00
$F^\#$	372.52	$F^\#$	367.92	$G^b/F^\#$	369.99
F	348.83	F	348.83	F	349.23
E	331.11	E	327.03	E	329.62
E^b	310.08	E^b	313.96	$E^b/D^\#$	311.13
D	294.33	D	294.33	D	293.66
$C^\#$	279.39	$C^\#$	272.54	$D^b/C^\#$	277.18
C	261.63	C	261.63	C	261.63

Wind instruments: waves in pipes

- Waves in pipes are important for musical instruments, ventilation ducts, and other applications
  - In a rigid-walled pipe with R << λ , sound propagates as plane waves, similar to longitudinal waves in solid bars
  - For rigid-walled volumes with R > λ , standing waves can appear: resonances

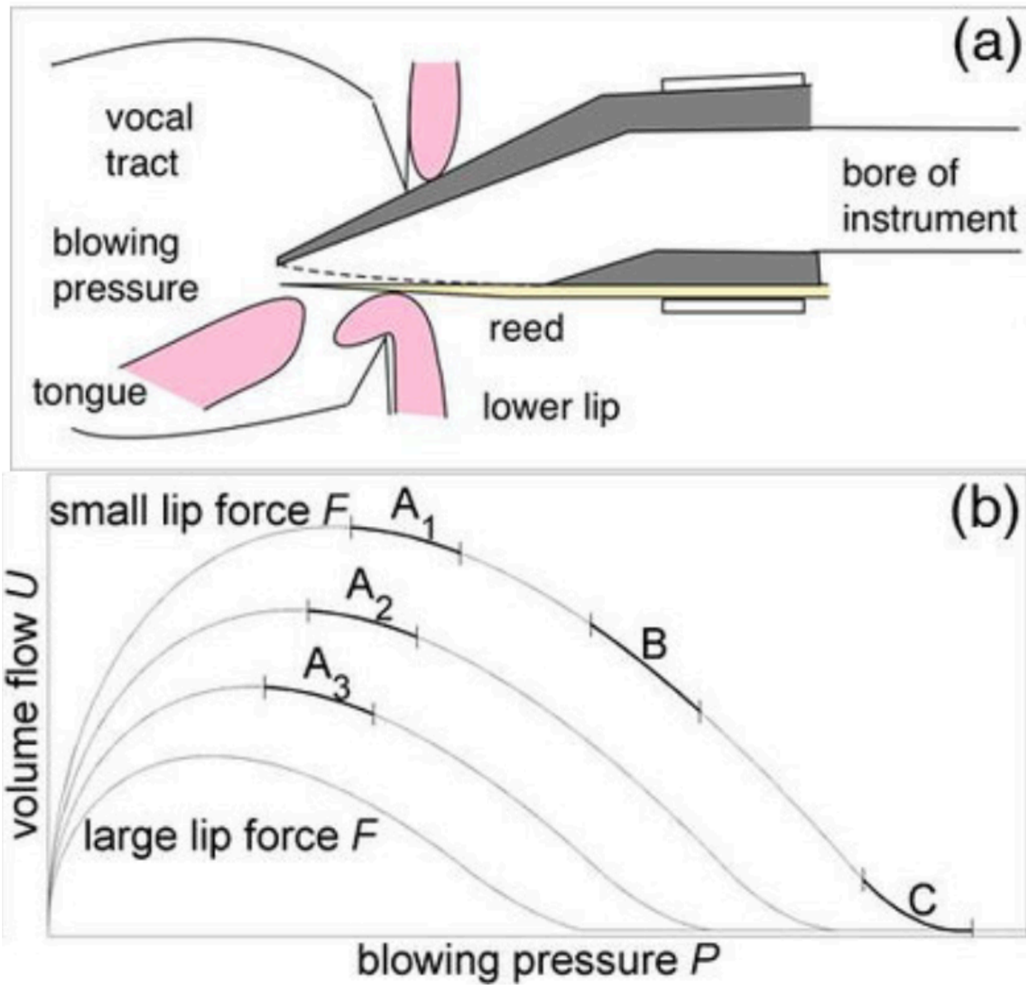
Pressure and displacement are out of phase:
highest p → no motion



- In musical instruments, resonant behavior is affected by
  - Shape of cavity (straight organ pipe, flared brass horn)
  - Material of instrument (rigid brass, or wood)
  - Nature of driver (lips, reed)
  - Perturbations due to ports or vents (finger holes, valves)

Wind instruments = Driven tubes

- Driver = mouthpiece + embouchure



clarinet mouthpiece and embouchure

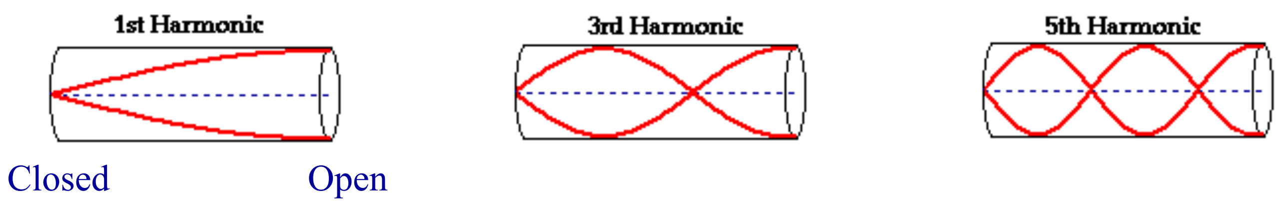
Proceedings of ACOUSTICS 2016, Brisbane, Australia, *Some secrets of good musicians: physics controlling articulation and timbre in reed instruments,* Weicong Li, et al, U. Sydney

Results of simple model of the flow U past the reed produced by blowing pressure P at different values of the force F applied by the lip

Musical instruments: organ pipes and horns

- *Organ pipes* often have one closed and one open end

- Open end must be a pressure node (P_o=atm → p=0), closed end must be anti-node (displacement=0)
    - Opposite for displacement of particles: open end = antinode (ξ = max)
  - So L must be multiple of *half* of $\lambda/2$: $L=N(\lambda/4)$
    - But if N=even number, we'd get *two* nodes: so N = odd # only



- So resonant harmonics are $L=\lambda/4$, $3\lambda/4$, $5\lambda/4$ … (1st, 3rd, 5th…)
- Resonant frequencies $f = c / \lambda$ → $f_n = n\, c / 4L$, n=1,3,5…

For air, c=343 m/s → f_n = 86 (n / L), n=1,2,3…

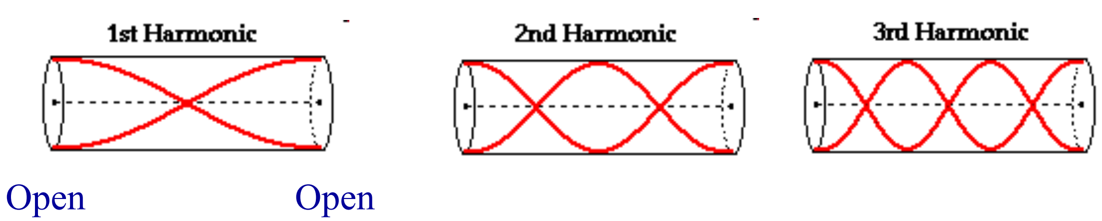
Example: B♭ trumpet has L=1.4m→ f_n = 61, 85, 306 Hz

- We can imitate organ-pipes by blowing across end of a bottle
  - Add water in bottle to change fundamental frequency

Open ended pipes

- Some instruments have both ends open: folk *flutes* (panpipes, Asian flutes), didgeridoo, etc

- Now both ends must be pressure nodes (displacement anti-nodes)
  - L should be integer multiple of $\lambda/2$: $L=n(\lambda/2)$
    - But now n=even number works also
  - So frequencies are same as for guitar strings

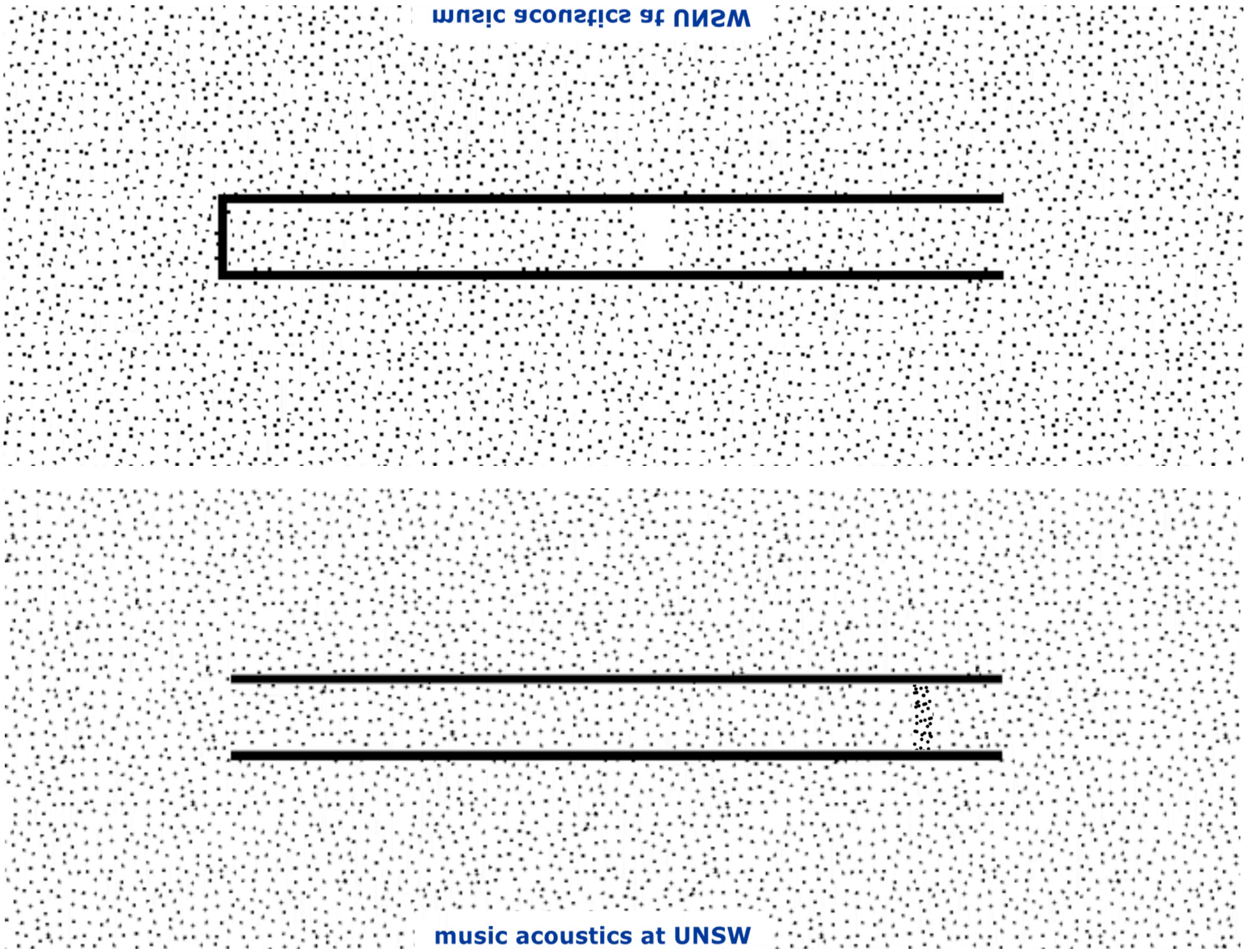


- Resonant harmonics are $L=\lambda/2, 2\lambda/2, 3\lambda/2$ … (1^{st}, 2^{nd}, 3^{rd}…)
- Resonant frequencies $f = c / \lambda \rightarrow f_n = n\, c / 2L$, n=1,2,3…

For air, c=343 m/s $\rightarrow f_n = 172\,(n / L)$, n=1,2,3…

Example: Didgeridoo with L=1.4m $\rightarrow f_n$ = 122, 244, 366… Hz

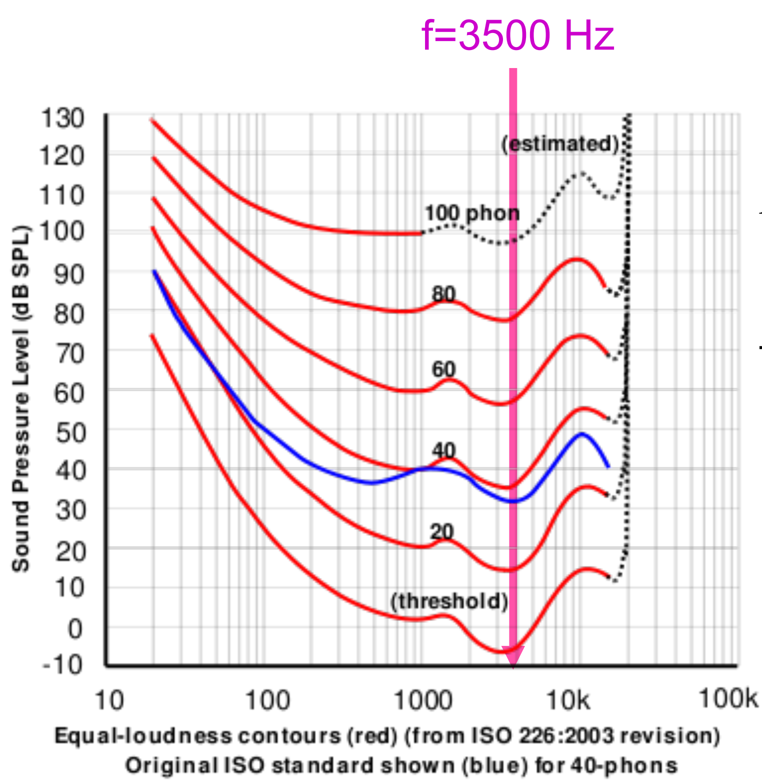
Open-closed and open-open tubes



- Brass and woodwind instruments work like organ pipes
  - Use *valves* to change effective length (brass), or impose an antinode somewhere inside (woodwinds)
  - Your ear canal is an example of a closed-end pipe

Example

- If the first harmonic of the human ear canal is at f=3500 Hz, and we model the ear canal as a simple organ pipe, how long must it be?

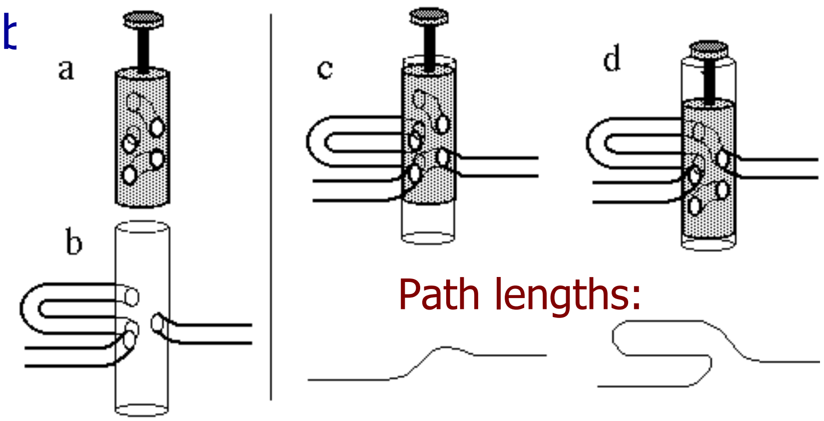


$$L_1 = \frac{\lambda_1}{4}, \quad f_1 = 3500 Hz = \frac{c}{\lambda_1} = \frac{c}{4L_1}$$

$$\rightarrow \quad L_1 = \frac{c}{4f_1} = \frac{(343 m/s)}{4(3500 Hz)} = 0.025 m \quad (1 \text{ inch})$$

Brass instruments

- Trumpets
  - B ♭ trumpet is most common (L=1.4m), C trumpet (L=1.3) used for orchestral music
    - Problem: cylindrical tube's resonances are too far apart to cover the scale
  - Valves increase L when pressed, lowering the pitch.
    - 1^{st} valve lowers the pitch b
    - 2^{nd} valve by 1 semitone
    - 3^{rd} valve by 3 semitones

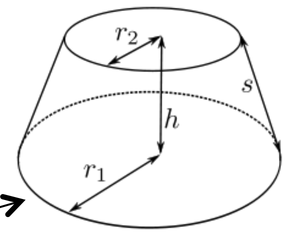


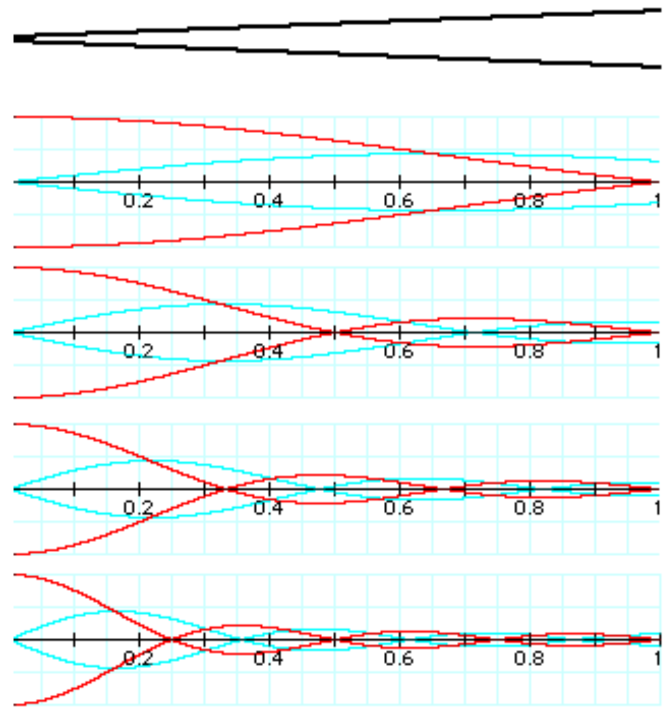
Valves make the trumpet "fully chromatic" = able to play all twelve tones of the scale

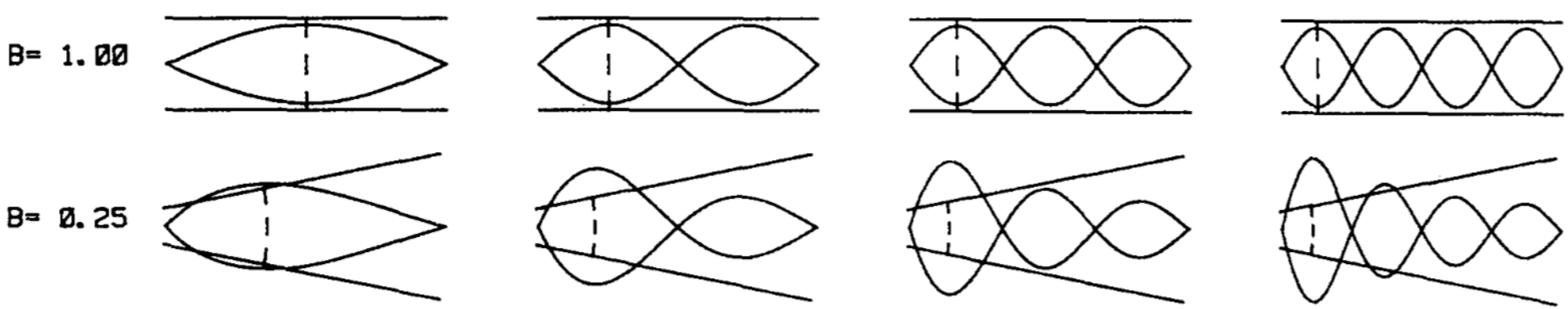
  - Problem: sound output of from unflanged tube is not very loud
    - Add a bell = conical flange on end of tube

Trumpet = Pipe + cone

- Bell on a trumpet or saxophone changes acoustics
  - Cylinder → plane waves,
  - Cone → spherical waves
    - Bell = frustrum (truncated cone) attached to cylinder
  - Standing wave pattern is changed
    - Effect depends on B=ratio of min/max radii of frustrum
    - First 4 SW patterns for B=1, 0.25:

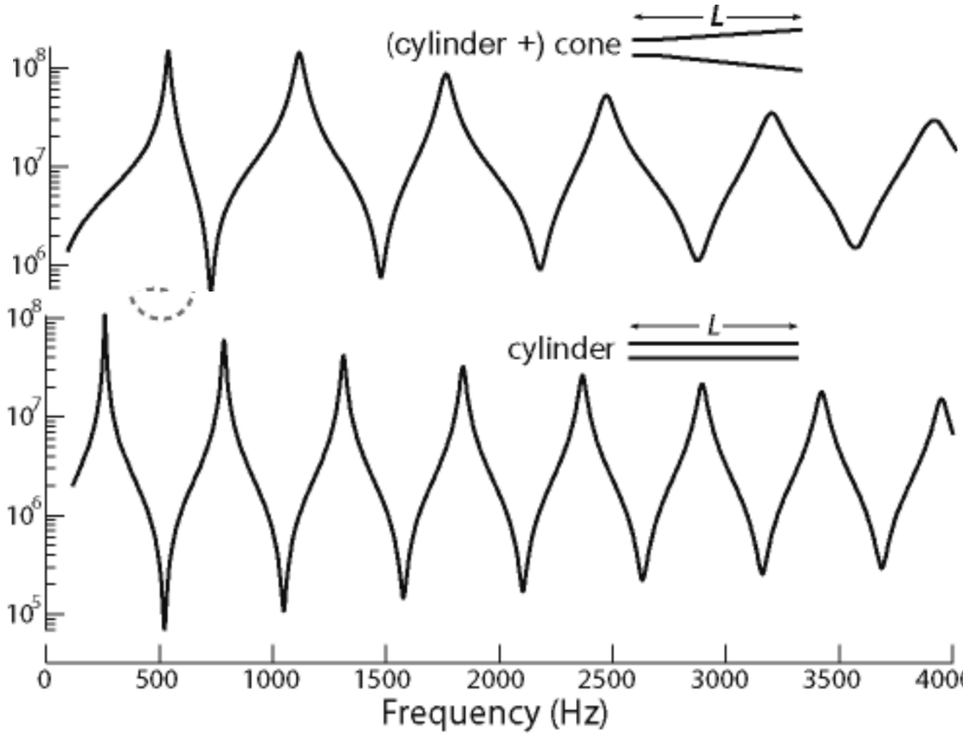




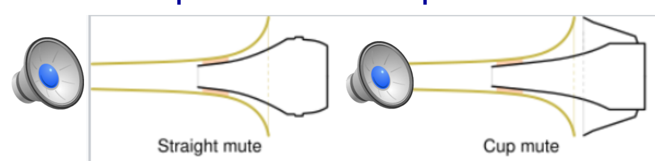


Ayers, et al, Am.J.Phys. 53:528 (1985)

Trumpet: bell, mutes, embouchure



- Bell shifts resonant f's up
  - Radiates high f's well
  - "Brassy" sound
  - Increases efficiency of sound propagation at open end
- Mute reduces high f's
  - Affects timbre as well as loudness
  - Less effect on 1—3 kHz
    - Corresponds to voice
    - Muted horn sounds "human"
      - Straight mute = highpass
      - Cup mute = bandpass



Player + trumpet =two resonators in series

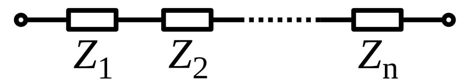
- Upstream is vocal tract + lips, downstream is instrument bore
  - Bore resonances are higher Q and dominate
- Tongue placement and changes in mouth configuration affect intonation

"details of how this works are subtle and not yet understood"

- *J. Wolfe, U. NSW, Australia*

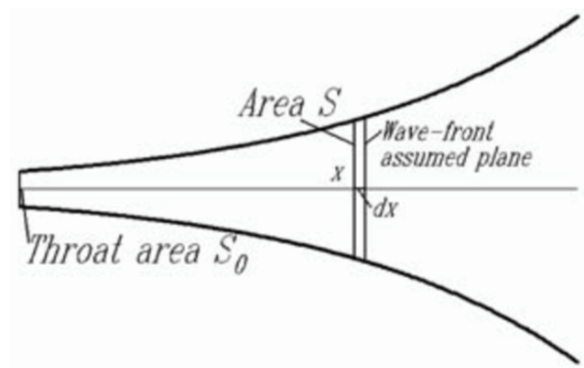
Exponential horn

- Simple conical bell shape is rarely seen
- Abrupt change in Z at each end of cone causes reflections
  - To eliminate, need $\lambda > 2r$, would require r~2m around 100 Hz



- Gradual impedance change → series of increasing-slope segments
  - $Z_{tube} \sim 1/\text{area} \sim 1/r^2$ → let $Z_k \sim \sqrt{(Z_{k-1} Z_{k+1})}$ = geometric mean Z
- Minimum total Z (sum of segments) → $S(x)=S_0\exp[(x-L_0)/L_0]$

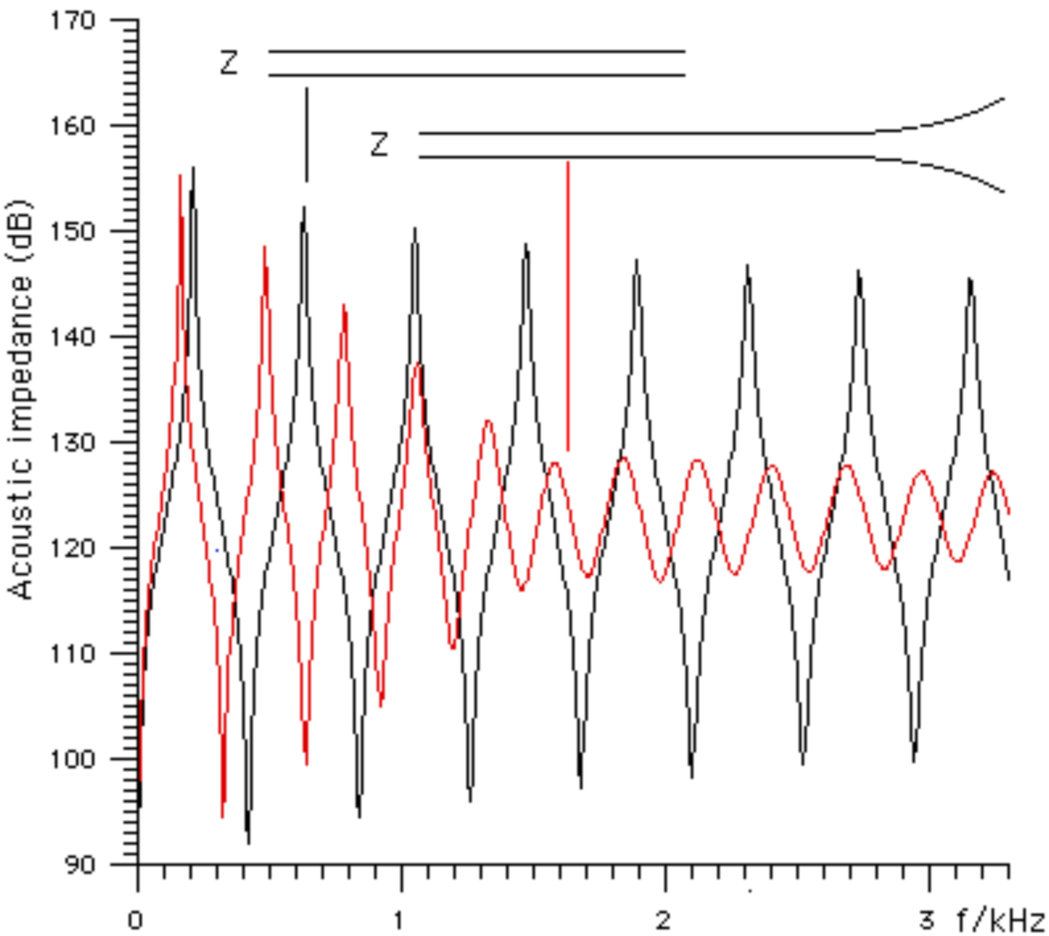
S_0 = mouth area, L_0 =length of horn, x = distance from throat



grc.com/acoustics/an-introduction-to-horn-theory.pdf

Acoustic impedance with/without bell

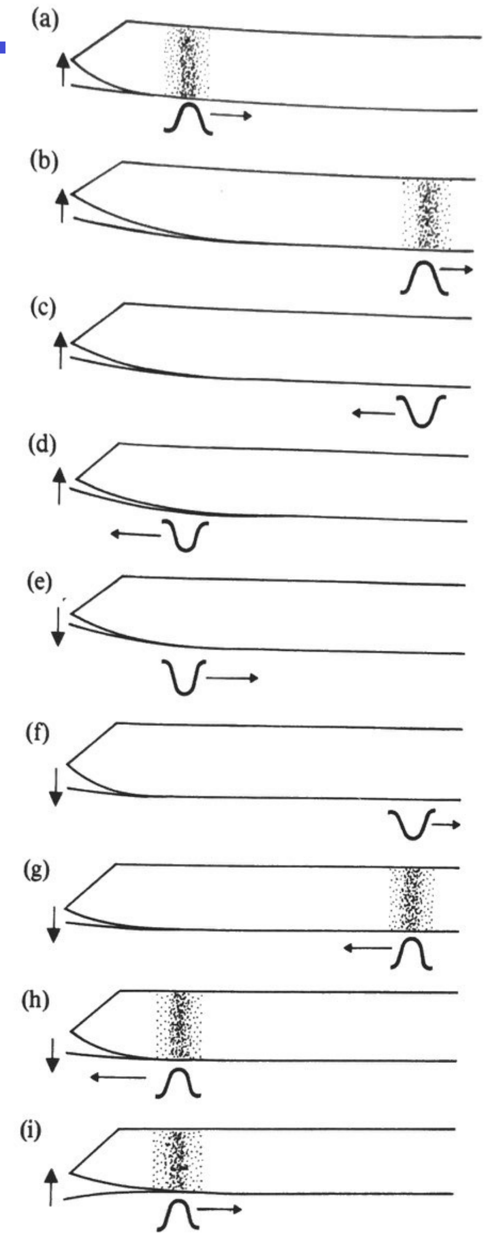
- Impedance of tube with and without exponential horn
  - Shifts resonant frequencies, and lowers Q for higher harmonics



- single/double reed instruments resonate at Z *maxima*
- Air reeds like flutes resonate at Z *minima* (open mouth hole)

Woodwind instruments

- Woodwinds have several types of driver
  - Clarinet, saxophone (single reed),
  - oboe, bassoon (double reed),
  - flutes (“air reed”)
- Resonances in tube are governed by opening/closing tone holes
  - Sound radiates from open tone holes as well as bell
- Pipe-reed system
  - Reed opens →puff of air, wave reflects off open end (inverted) [a,b,c in figure →]
  - Reed closes when negative p pulse arrives: re-reflects (inverted) [d, e]
  - Second reflection from bell end (positive p) opens reed [f, g, h]
    - start again with next air puff
  - Alternative: with high embouchure pressure, reed closes/opens/closes in 1 round-trip of wave: f → 3f



Woodwinds

- Resonant mode frequencies: pulse makes round trip each half-cycle of reed vibration :

$$T_1 = \frac{4L}{c}, \quad f_1 = \frac{1}{T_1} = \frac{c}{4L}, \quad L = \text{ effective length of tube}$$

$$f_1 = 3f_1 \quad \text{higher modes are} \quad f_n = nf_1 \quad n = \text{odd integers}$$

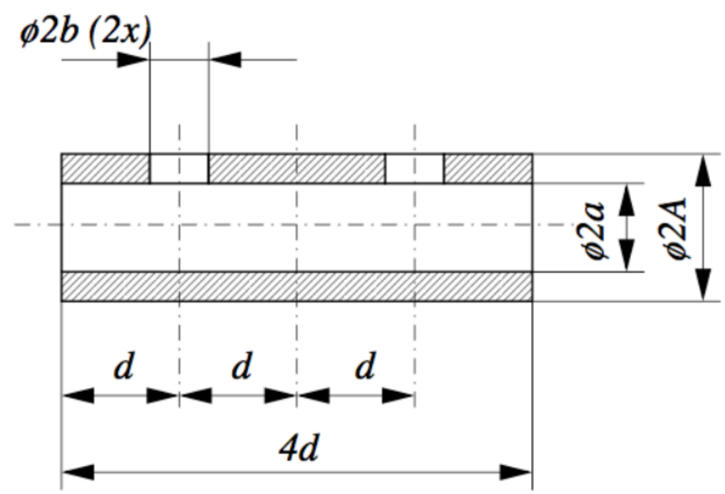
- Tone holes change effective L of tube
  - Array of open holes = tone hole lattics → resonant structure
  - Lattice create cutoff f → harmonics above this are suppressed
  - Cutoff f is (from Benade*)

(clarinet has L~0.6m)

$$f_c = 0.11\frac{b}{a}\frac{c}{\sqrt{d(t+1.5b)}}$$

t = A-a

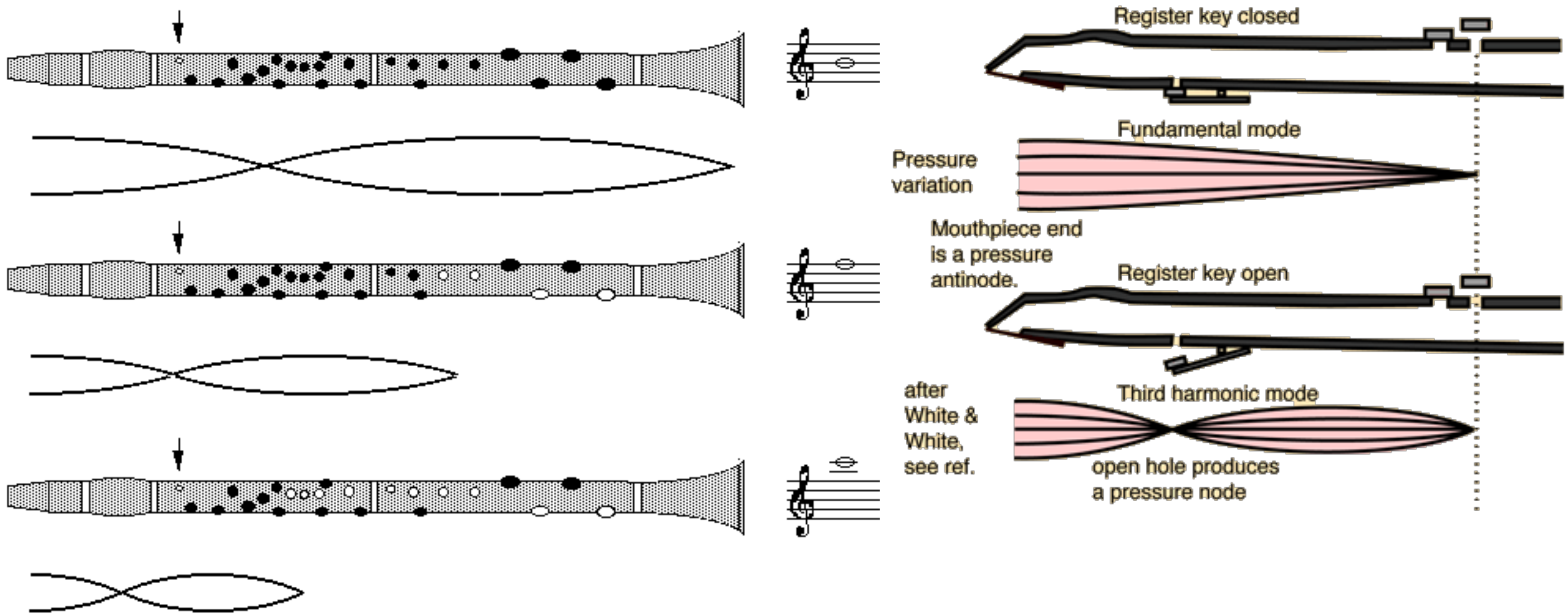
c = sound speed



* A. Benade, *Fundamentals of musical acoustics,* 1976

Woodwinds

- Register holes = openings to kill fundamental and shift range up
  - Put the opening to P_{atm} at a node of the higher harmonic – little effect on it, disrupts fundamental
    - Must use compromise location to cover each register's range efficiently
      - Clarinet has 1 register (or "speaker") hole, saxophone has 2, oboe 3



- Famous use of mouth and tone holes with clarinet: Gershwin's Rhapsody opening

See 0:00, 1:45, 2:40, 4:35 in https://www.youtube.com/watch?v=45XeZOfuc9c

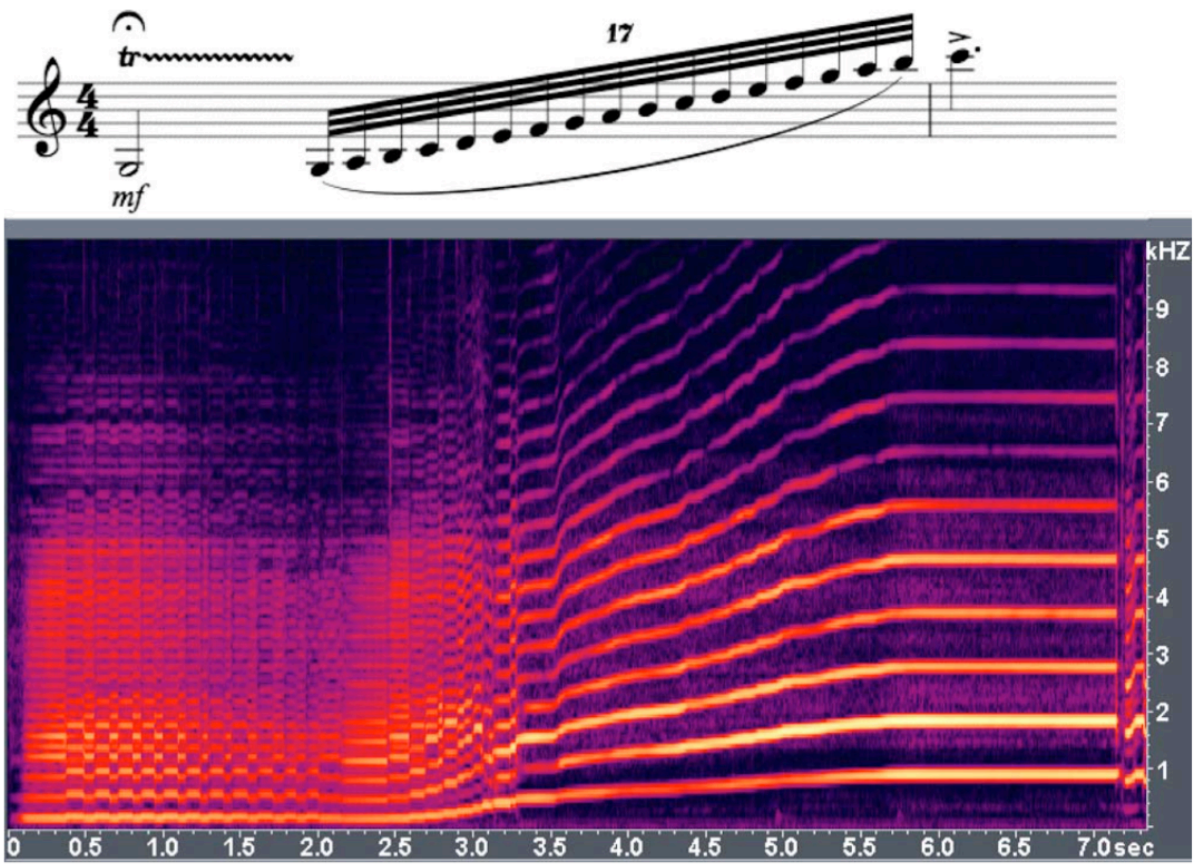
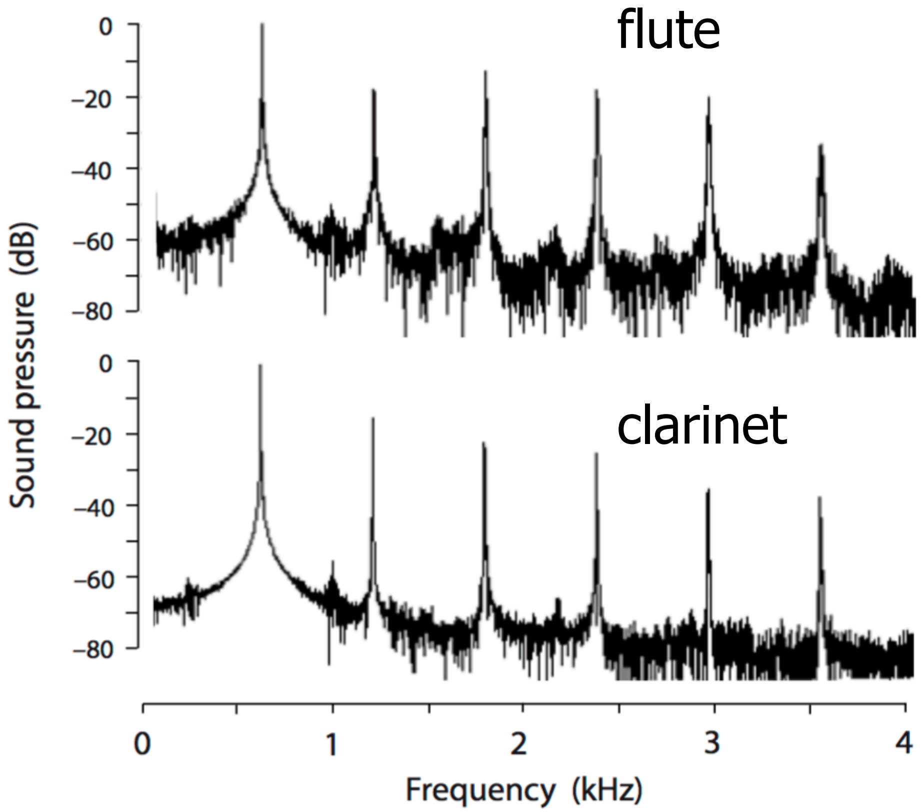


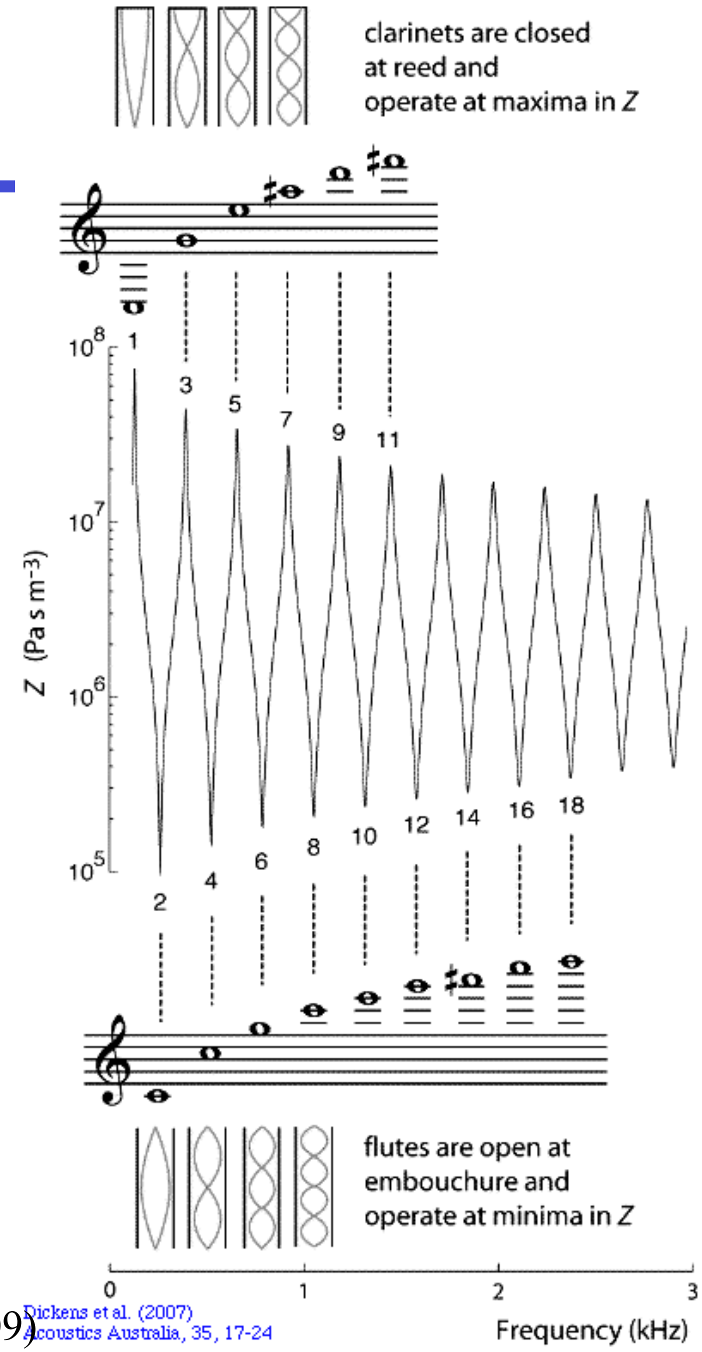
FIG. 1. (Color online) The opening of Gershwin's Rhapsody in Blue. The upper figure shows the 2.5 octave run as it is written—but not as it is usually played. In traditional performance, the last several notes of the scale are replaced with a smooth *glissando*. The lower figure is a spectrogram of such a performance. The opening trill on G3 (written, 174 Hz) is executed from 0 to 2 s and followed by a scale-like run at 2.5–3.5 s that becomes smooth pitch rise from C5 (466 Hz) to C6 (932 Hz) at 3.5–5.7 s.

Pitch bending and glissandi on the clarinet, J. Chen et al, JASA 126:1511 (2009)

Clarinets vs flutes



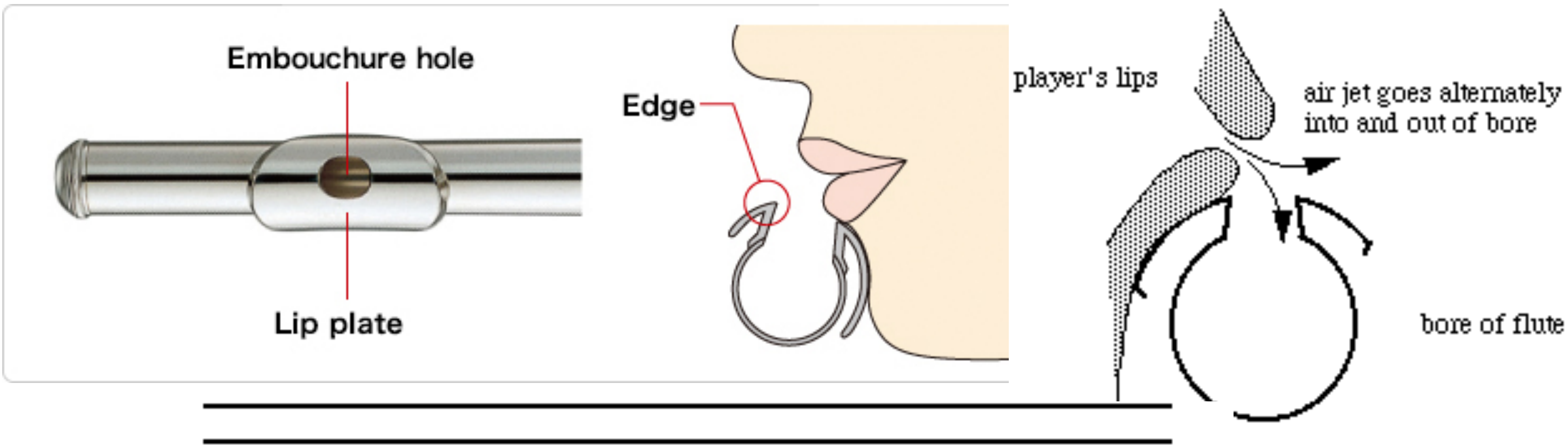
Sound power spectra for D5 played on a flute (upper) and a clarinet (lower)

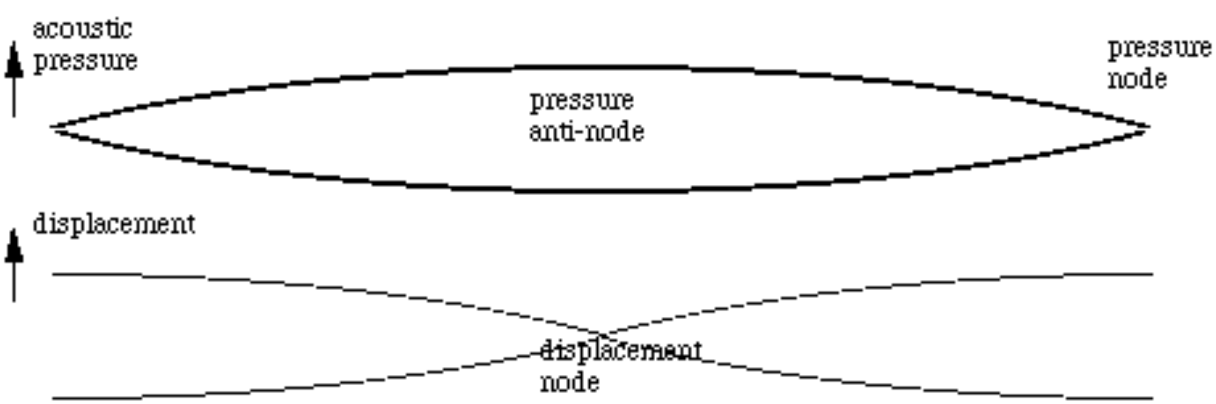


Clarinet acoustics, Dickens et al, *Acoustics Australia*, 35:17 (2009)

Flutes

- Flute = open-open tube with excitation by air blown over embouchure hole, which has sharp edge opposite lip plate



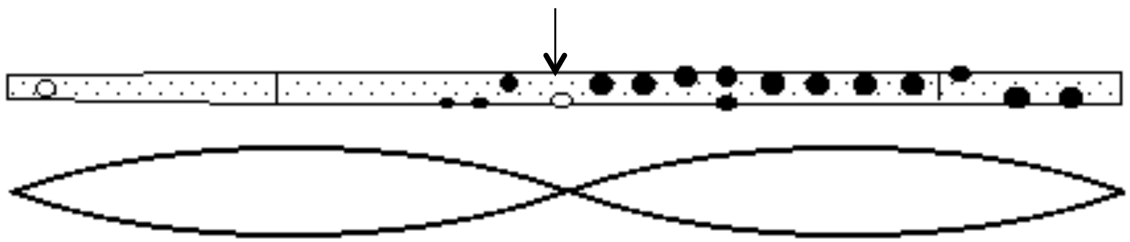


Fundamental

Flutes

- Flute requires more complex embouchure work by player
  - Air jet (20 to 60 m/s) from lips strikes the sharp edge of the hole
  - Perturbation of jet → flows into or out of the embouchure hole
  - Resonant waves in the tube make air flow into and out of the embouchure hole
  - Player matches f of the note desired, so jet flows in and out of the hole in phase with tube wave → sustained note.
  - For high notes, player increases pressure (increases the jet speed) and moves the lips forward to shorten the distance to the edge of the embouchure hole.

Tone holes make L_{eff} shorter

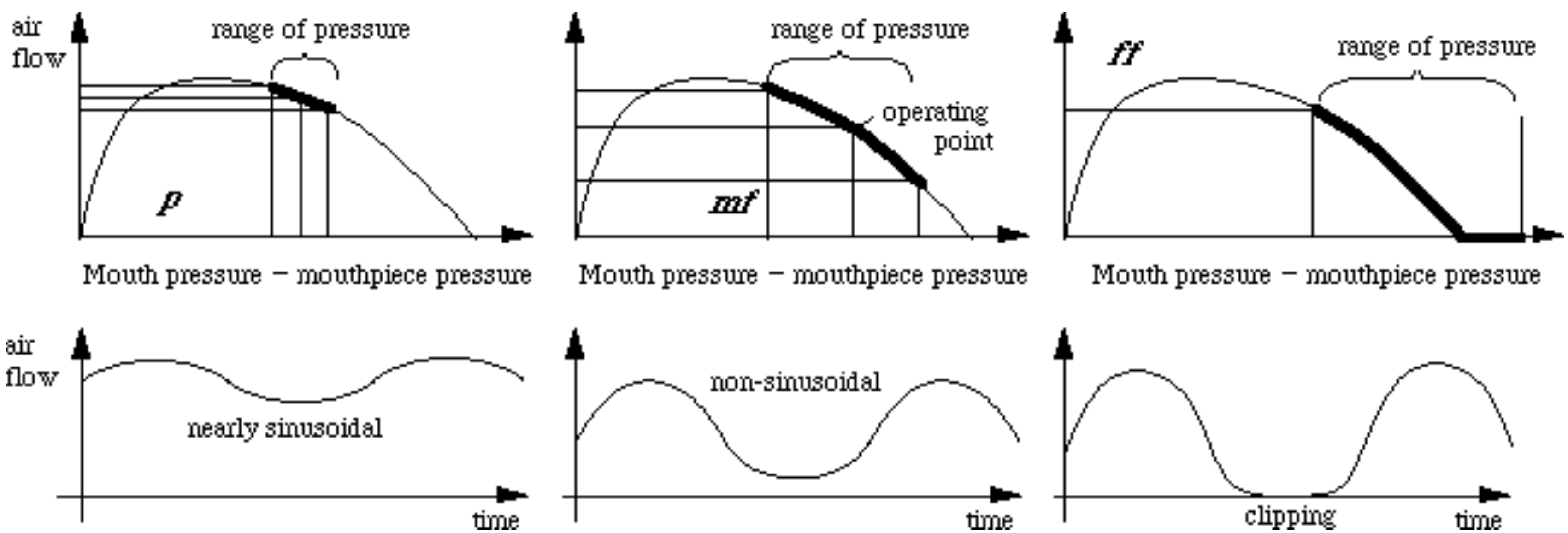


Register holes kill odd harmonics so pitch jumps

C4→C5

Saxophones

- Brass “woodwind”, patented in 1846 by Adolphe Sax
  - Single reed of the clarinet +
  - Conical bore and fingering patterns of the oboe
    →completely new tonal qualities.
- Pressure changes timbre
  - Moderate → sine wave, “pure”
  - Higher → non-sinusoidale → many higher harmonics
  - Highest → clipping → brighter timbre



Saxophone spectra

- Conical-bore instruments like sax have weaker first partial than cylindrical
- Saxophone's bell has little effect except for higher harmonics