PHYS 536

R. J. Wilkes

Session 15

Transducers

2/21/2023

Course syllabus and schedule

See : http://courses.washington.edu/phys536/syllabus.htm

Session	date	Day	Readings:	K=Kinsler, H=Heller	Topic
8	26-Jan	Thu	K: Ch. 7	H: Ch. 7	Absorption losses; Pulsating spheres and simple sources; pistons and dipoles; Near field, far field; Radiation impedance; Waves in pipes *UPDATED BELOW HERE:*
9	31-Jan	Tue	K. Ch. 8-10	H: Ch. 13	Rectangular cavities; Helmholtz resonators; Resonant bubbles; Acoustic impedance; physical acoustic filters; Doppler effect; Interference effects
10	2-Feb	Thu	K. Ch 9	H: Chs. 23-25	Musical acoustics: pitch, musical tones and frequency; timbre; beats
11	7-Feb	Tue		H: Chs. 16, 18	Musical instruments: winds and string instruments
12	9-Feb	Thu		H: Chs. 17, 19	Musical instruments: piano, human voice REPORT 1 PAPER DUE by 7 PM; REPORT 2 PROPOSED TOPIC DUE
13	14-Feb	Tue	K. Ch. 11	H: Ch. 21	Human hearing: the inner ear; pitch perception; acoustics of speech
14	16-Feb	Thu	K. Ch. 12	H: Chs. 21-22	Decibels and sound level measurements Environmental acoustics and noise criteria; industrial and community noise regulations; noise mitigation;
15	21-Feb	Tue	K. Chs. 13-14	H: Chs. 27-28; Ch. 6	Room acoustics; Transducers for use in air and water: Microphones and loudspeakers; hydrophones and pingers; Underwater acoustics: sound absorption underwater, the sonar equation
16	23-Feb	Thu	K. Ch 15		Underwater acoustics applications: acoustical positioning, seafloor imaging, sub-bottom profiling; Course wrap-up: review
17	28-Feb	Tue			Student report 2 presentations
18	2-Mar	Thu			Student report 2 presentations
19	7-Mar	Tue			Student report 2 presentations
20	9-Mar	Thu			Student report 2 presentations. TAKE-HOME FINAL EXAM ISSUED
--	17-Mar	Fri			FINAL EXAM ANSWERS DUE by 5 PM

Class is over after you turn in your take-home exam. No in-person final exam during finals week.

Tonight (← session 15)

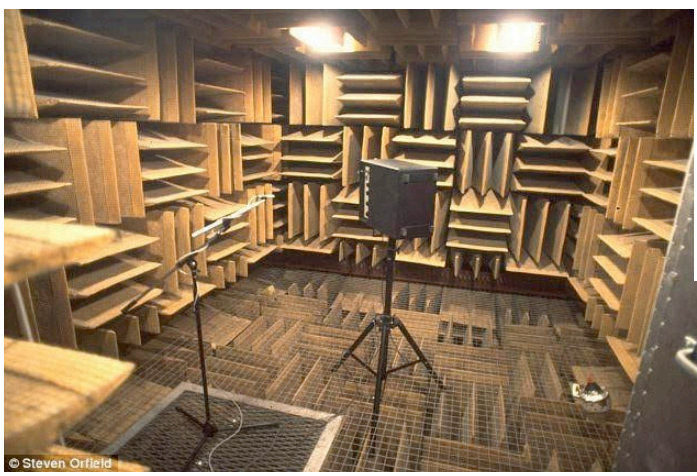
Announcements

End of term is creeping up – paper 2 is due **in one week**

- Everyone gives a brief (15 min max) presentation in class describing what you learned
  - You should submit your slides via Canvas assignments page on 2/28
    - You can tweak your slides before your presentation time, but for fairness your Canvas submission should be a complete draft
- We need to assign time slots over the 4 nights of presentations
  - Willing to go on the first night (2/28)? pls tell me by email ASAP!
    - Also tell me ASAP if you need a particular day/time for any reason
    - I'll have to arbitrarily assign all remaining slots (random order)
- On the night of your presentation slot
  - **You must email me your final slides as a pdf or ppt file**
    - Slides and notes must be sent by **6 pm on night you present**
  - I will share your screen – but if you do not have adequate setup, let me know and I will display your sides
  - You MUST have a microphone – please test your audio before class

Measuring very low SPLs

- The anechoic chamber at Orfield Laboratories in Minneapolis is "the quietest place in the world" (Guinness), with a background noise reading of –9.4 decibels – how can this SPL be measured?
  - Utter silence is very disorienting, can cause people to have trouble standing – people can't tolerate it more than 45 min
- Correlation technique: 2 identical mics face to face
  - Signal = p(t) + n(t)   (room SPL and mic noise)
  - Two identical mics: $s_1(t) = p_1(t) + n_1(t)$, $s_2(t) = p_2(t) + n_2(t)$
    - $n_1(t)$ and $n_2(t)$ are uncorrelated and cancel when you average:



www.gracey.co.uk/downloads

Microphones should be closer than a quarter wavelength. For lower frequencies, the microphones may be placed side-by-side.

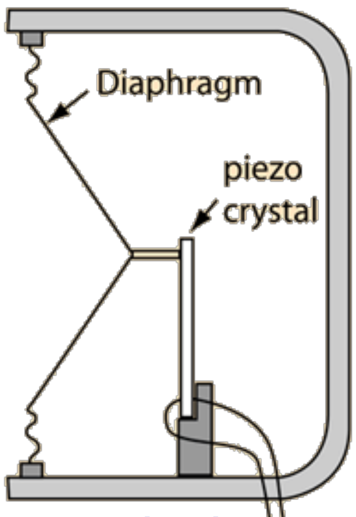
$$L = 10\log\left(\frac{\langle s_1(t)s_2(t)\rangle}{p_0^2}\right),$$

$\langle\ \rangle = \textit{average}, \quad p_0 = \textit{reference level}$

Transducers

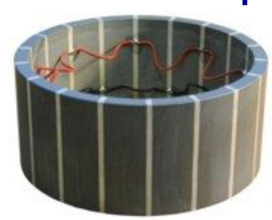
Transform electrical signals into sound waves, or vice-versa

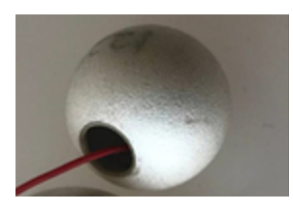
- Most common types: piezoelectric, “condenser” (capacitor), or electromagnetic (“dynamic”)



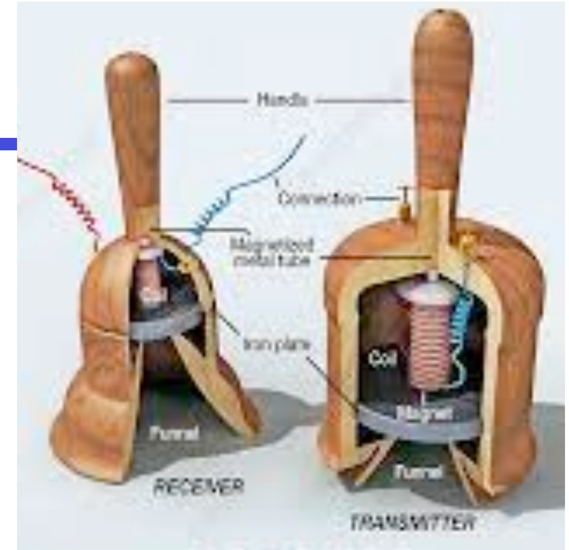
Principle	Strain Sensitivity [V/με]	Threshold [με]	Span to threshold ratio
Piezoelectric	5.0	0.00001	100,000,000
Piezoresistive	0.0001	0.0001	2,500,000
Inductive	0.001	0.0005	2,000,000
Capacitive	0.005	0.0001	750,000
Resistive	0.000005	0.01	50,000

  - Piezos: highly sensitive, low threshold, wide range of amplitude, insensitive to electromagnetic fields and radiation – but high Z
    - High impedance → matches vacuum tube inputs but not solid state electronics; high noise susceptibility
    - Useful for underwater transducers: ceramic piezo works well in high pressure environment

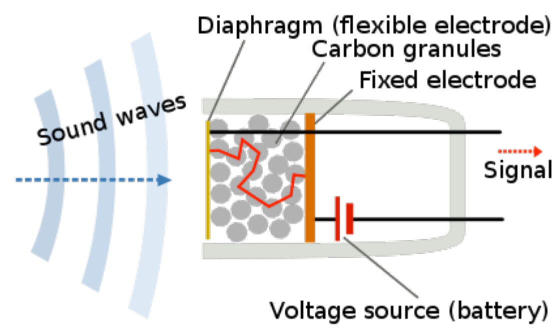


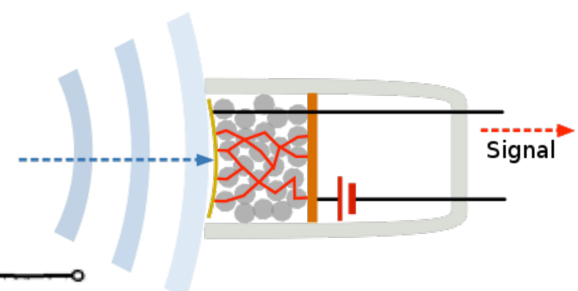


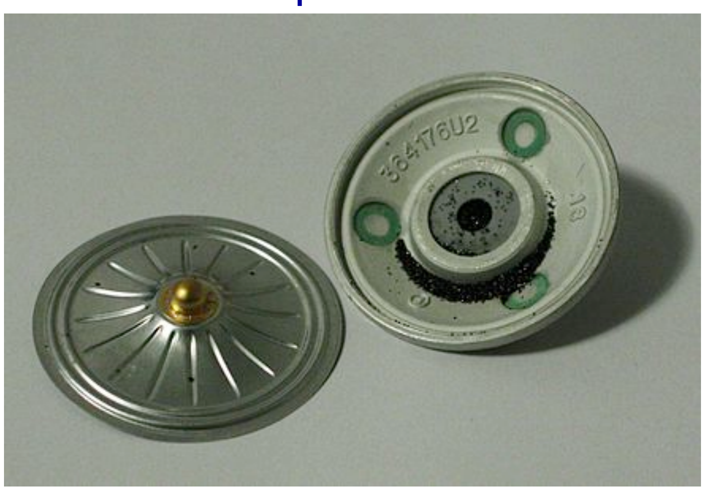
Early microphones



- Antonio Meucci invented an electromagnetic microphone in 1856 with moving a coil in a magnetic field – used for in-home communications!
- The carbon microphone was independently developed in 1878 by D. E. Hughes in UK, and E. Berliner and Thomas Edison in the US.
  - Carbon microphone = variable resistor acting on current from battery
    - Used as amplifiers in early telephone repeaters, allowing long distance phone before vacuum tube amplifiers
      - » Electromagnetic telephone receiver is mechanically coupled to a carbon microphone.







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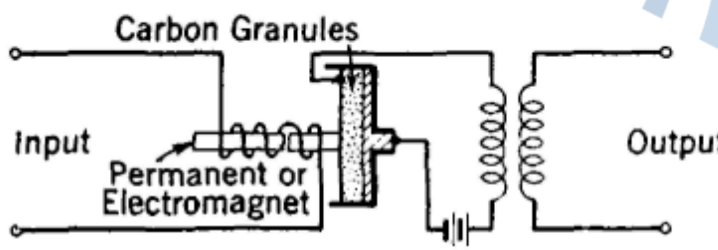


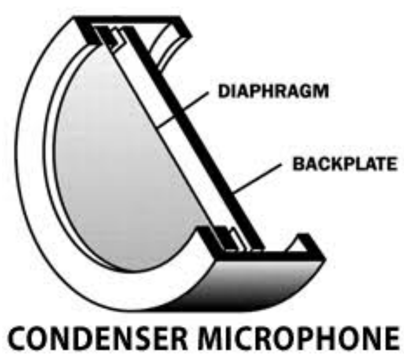
An early application for carbon mics

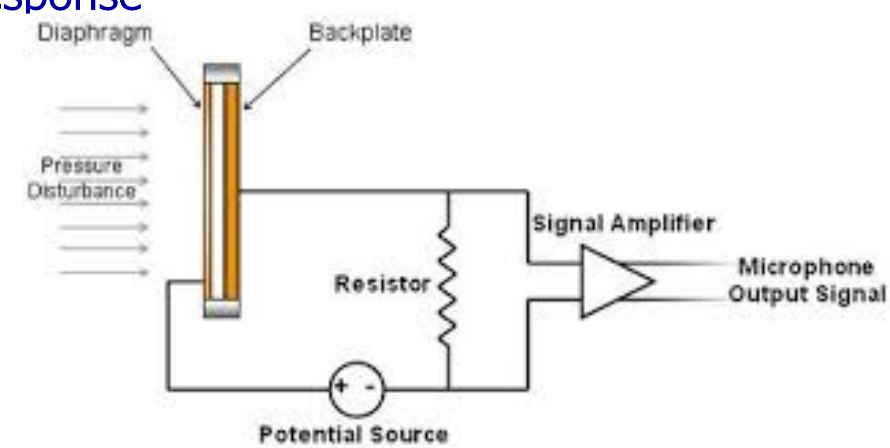
(Thanks to student report where I learned about this!)

- Théâtrophone (1881): telephone distribution system in Europe that let subscribers to listen to live performances over the telephone lines
  - Clément Ader, French engineer: in 1878 improved on the telephone invented by Bell, and established the first telephone network in Paris in 1880 (also contributed to aviation and gasoline engines)
  - Streaming live music 80 years before the internet
    - Also provided in-home access to news and information

Condenser mics

- Condenser microphones (E. C. Wente, 1916): low cost, sturdy
  - diaphragm acts as one plate of a capacitor, vibrations produce changes in the distance between the plates → varying C
  - The voltage across the capacitor reflect changes in capacitance.
  - DC-biased microphone: plates are biased with a fixed charge
    - Electret: material with quasi-permanent surface charge or dipole moment - no bias power required
  - RF condenser microphone: low voltage RF signal modulated by capacitance changes, yields a low-noise audio frequency signal with a very low source impedance.
    - absence of higher DC bias voltage allows RF mic diaphragms with less stiffness, wider frequency response

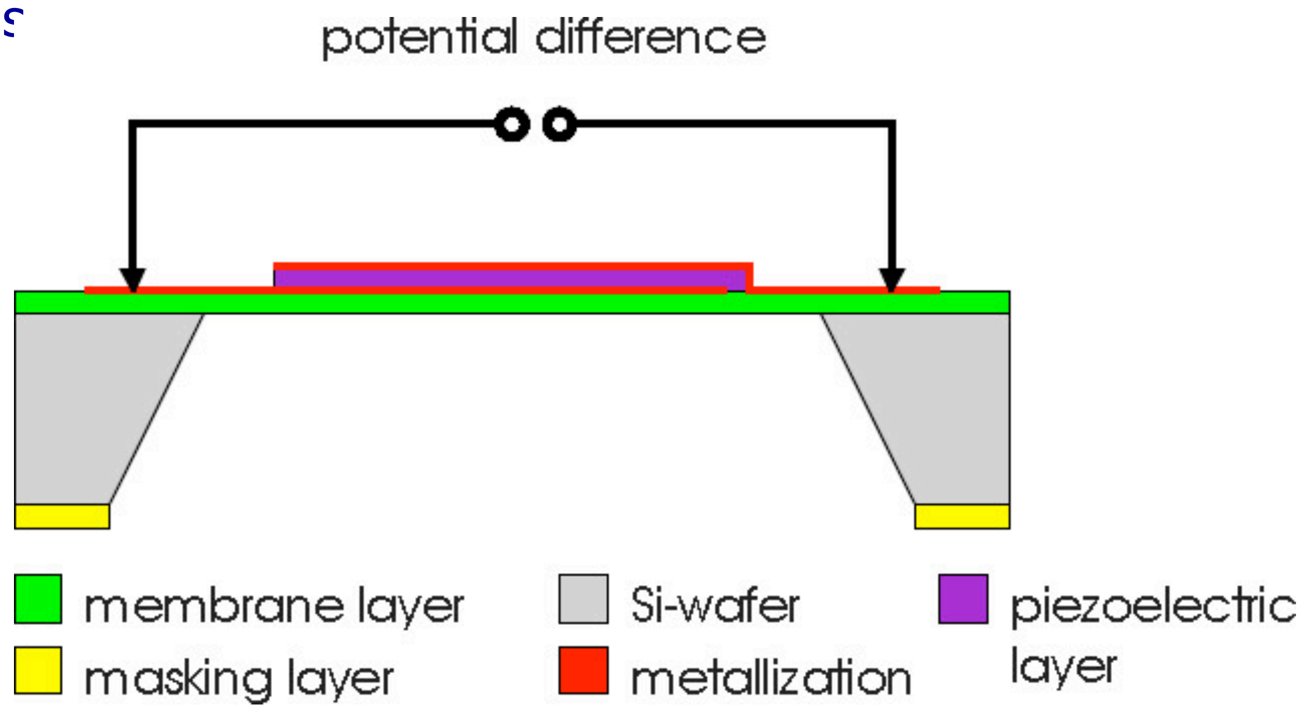




Piezo-electric mics

- Piezo element produces voltage difference with stress
  - Fabricated as single chip - example of a Micro-ElectroMechanical System (MEMS)
  - Durable and cheap, relatively large electrical output, can work directly off the vibrations on a surface (contact mic), no power source needed
  - Highly linear up to very high sound pressure levels (170 dB), simple construction, survive high temperatures, unaffected by dust – good for outdoor applications

Piezoelectric materials used in silicon microphones are ZnO, PVDF (polyvinylidene fluoride), and PZT (PbZrTi) ceramic



https://acoustics.org/pressroom/httpdocs/137th/fischer.html

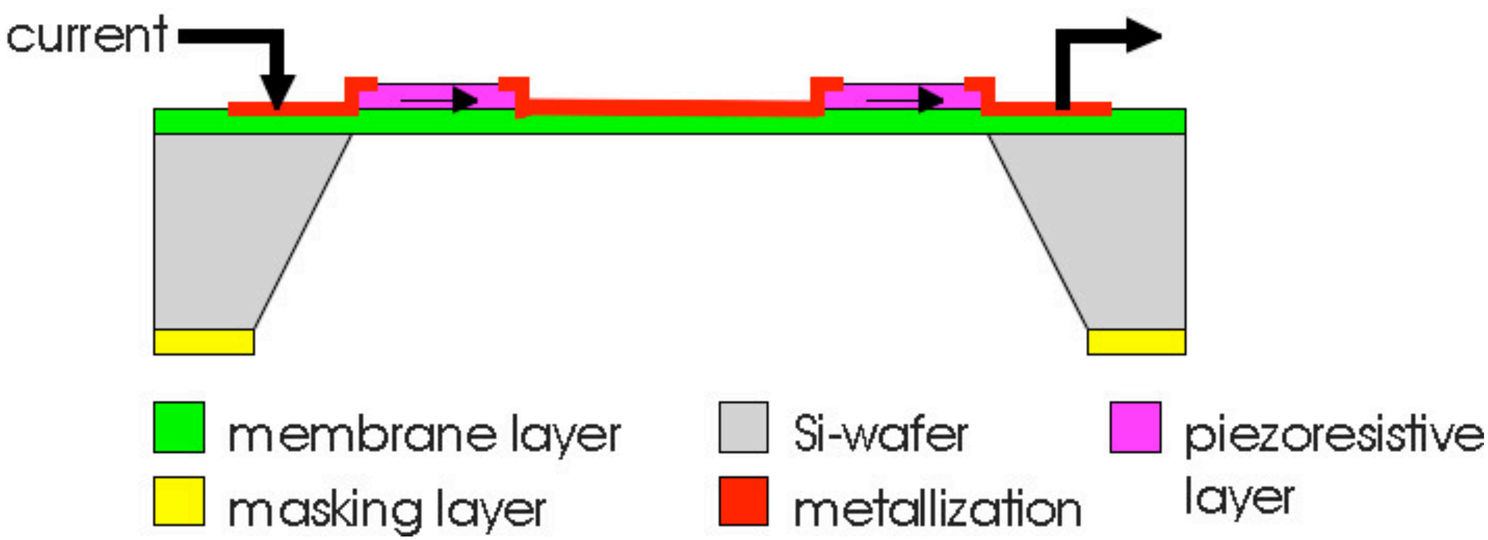
Piezo-resistive mics

(Differ from piezo-electric, which generate voltage when stressed)

- Piezo element varies resistance with stress: carbon mic updated!
  - Another example of a MEMS device
  - Low noise, high linearity up to extreme SPLs, simplicity, low cost
  - Polycrystalline or monocrystalline Si used for the piezoresistive layer

Typically four resistors are used in a Wheatstone bridge to increase the sensitivity

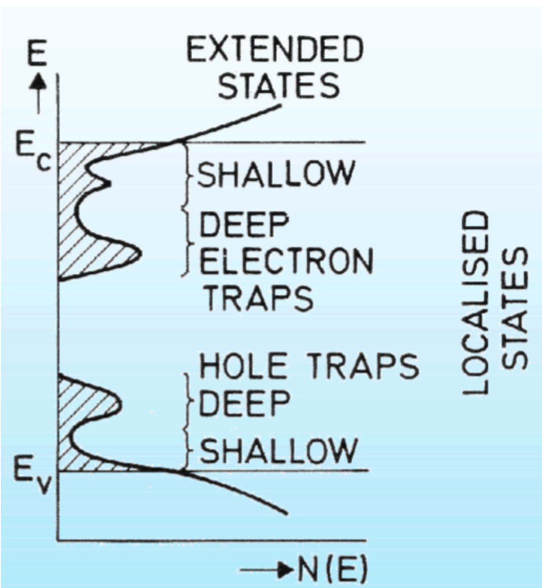
2 piezoresistive elements in series connection

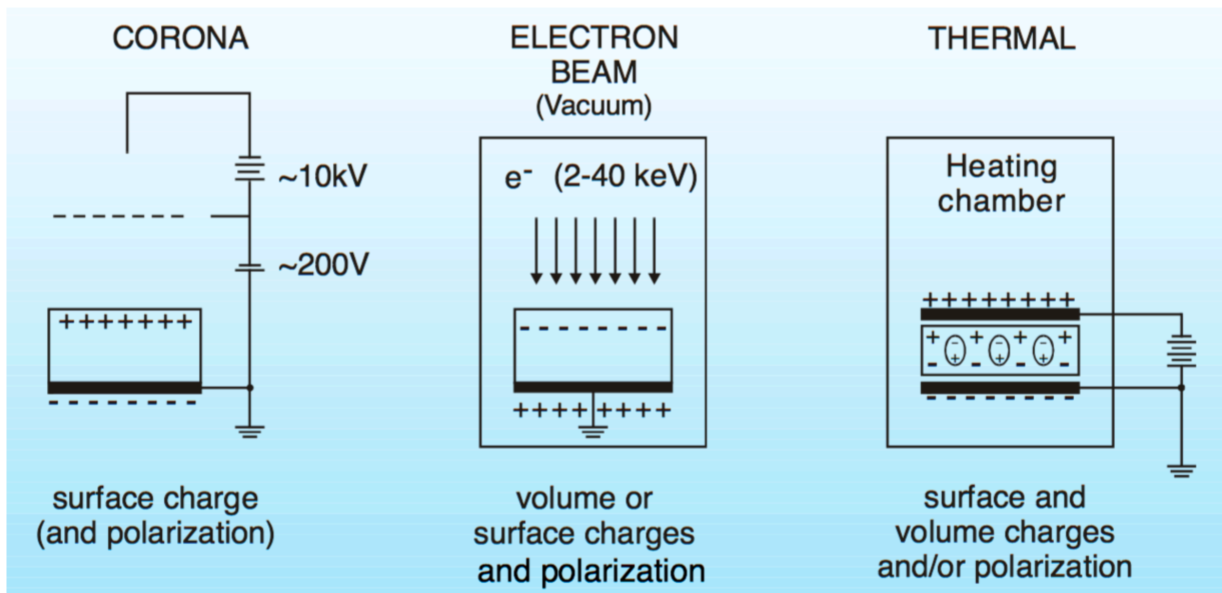


https://acoustics.org/pressroom/httpdocs/137th/fischer.html

Electrets

- Made from waxes, polymers or resins, by solidifying while in E field, or adding charge to surfaces
  - Quasi-stable charge trapping – lifetime = decades
- Real-charge or dipole electrets
  - Real-charge electrets which contain excess charge of one or both polarities, either on the dielectric's surfaces (a surface charge) within the dielectric's volume (a space charge)
  - Oriented-dipole electrets contain oriented (aligned) dipoles

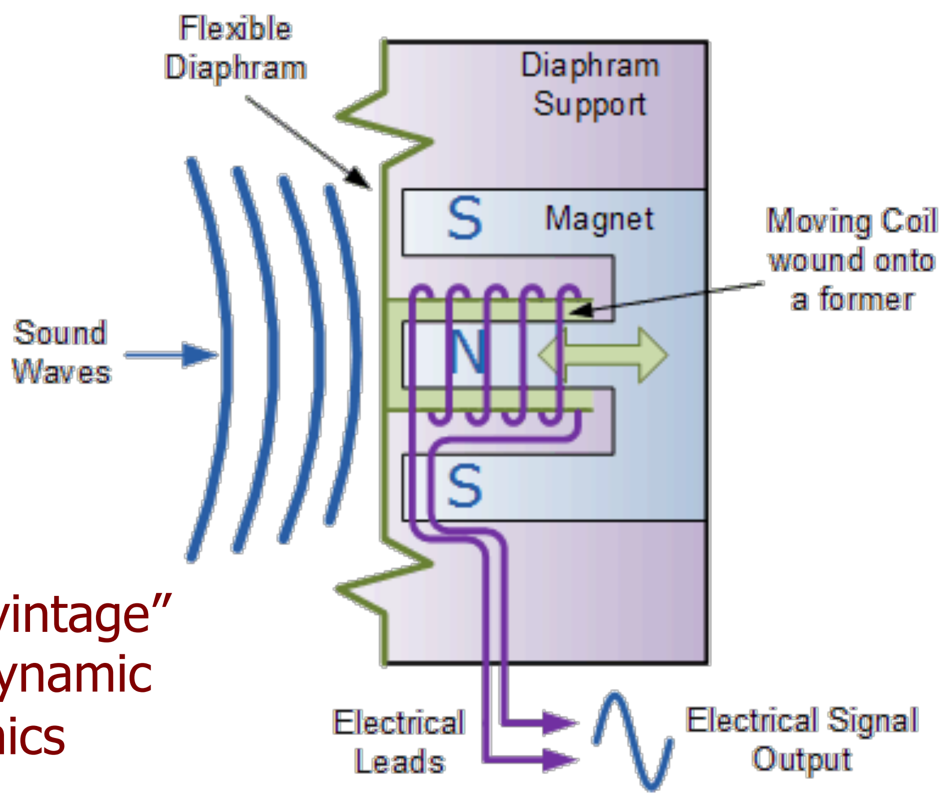




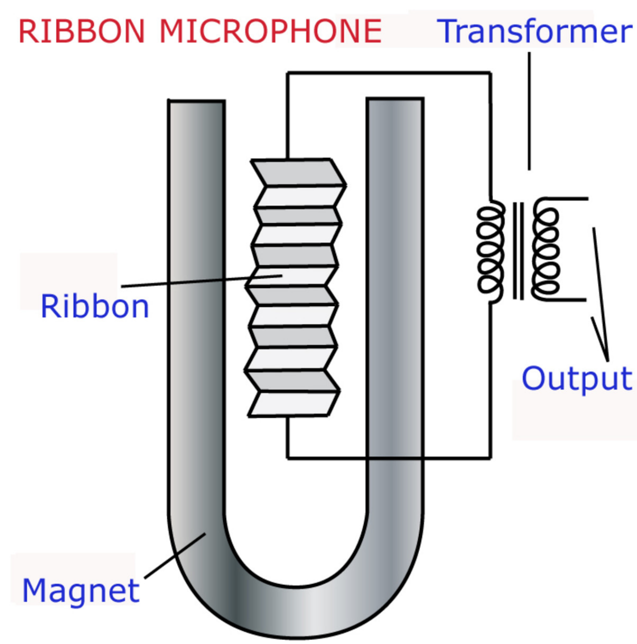
Dynamic microphones

- "Dynamic" (electromagnetic) transducers typically use permanent magnet and vibrating coil or foil to generate signal
  - Rugged, work best to record loud sounds.
  - output is considerably lower than for condenser. poorer high f response -- used for rock concerts, outdoor sound
  - Ribbon mics → high quality studio mics

"vintage" dynamic mics

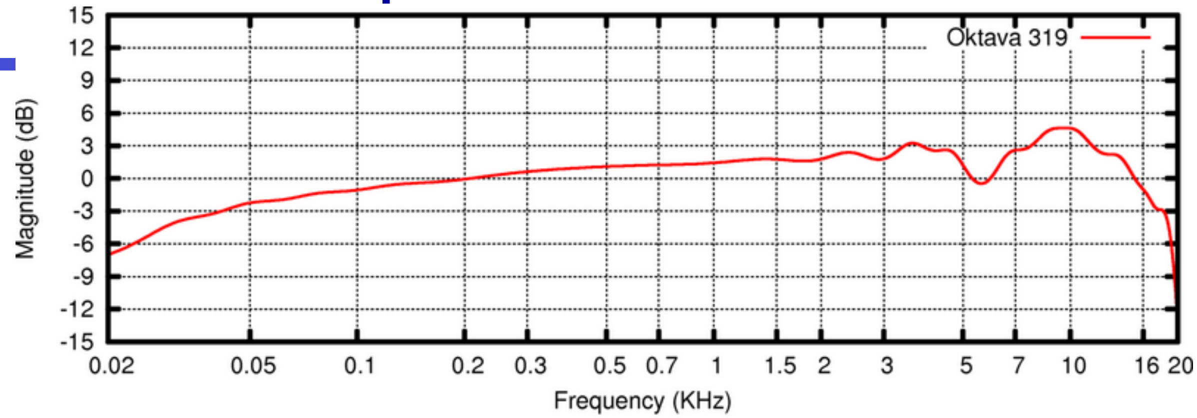


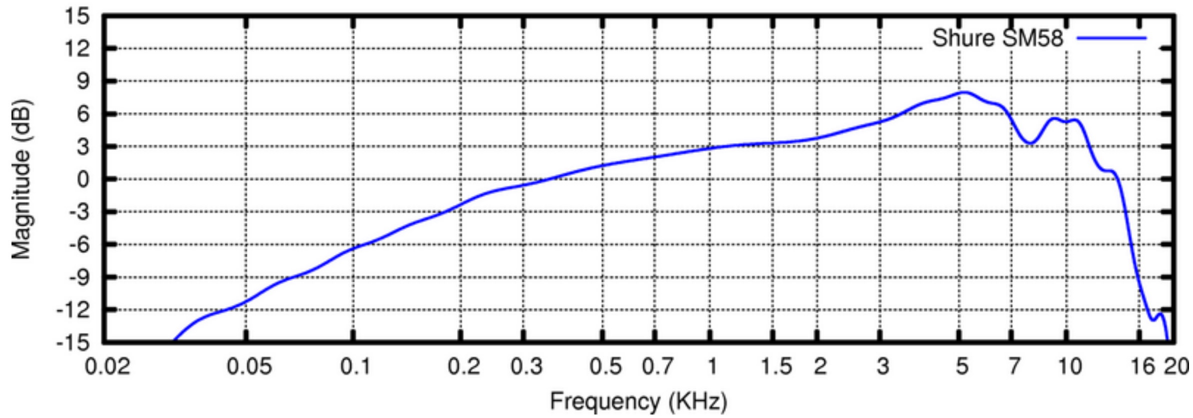
Ribbon mic uses metal foil instead of coil : better fidelity, higher cost, fragile

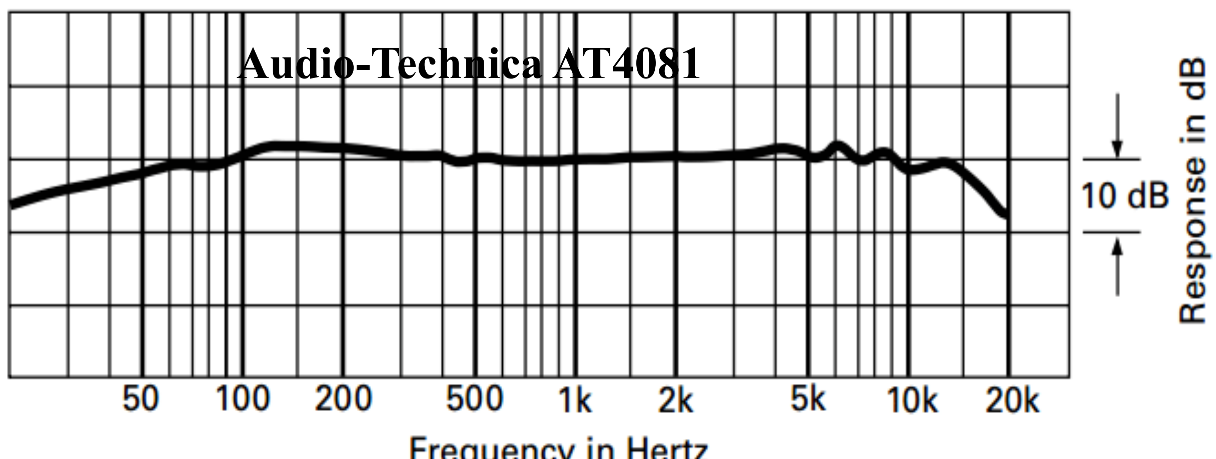


Dynamic microphones

- Comparison of frequency responses for different mic types
  - Electret condenser
  - Dynamic
  - Ribbon

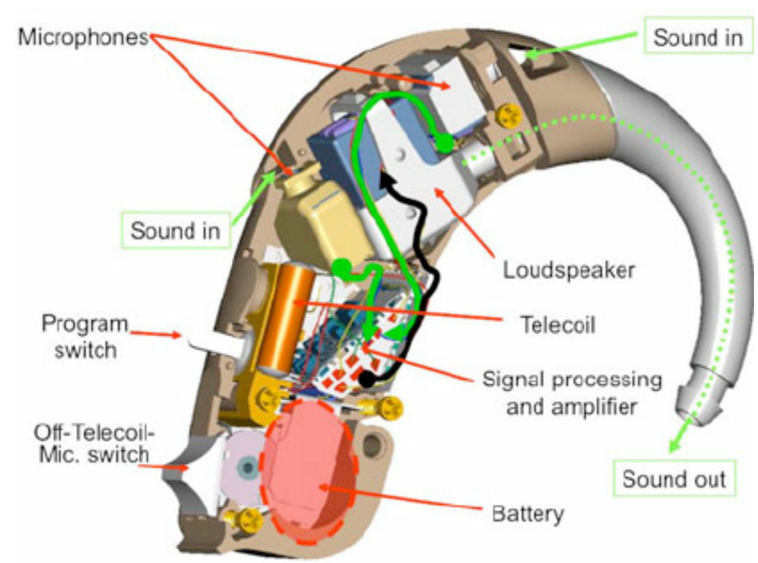






Application: hearing aids

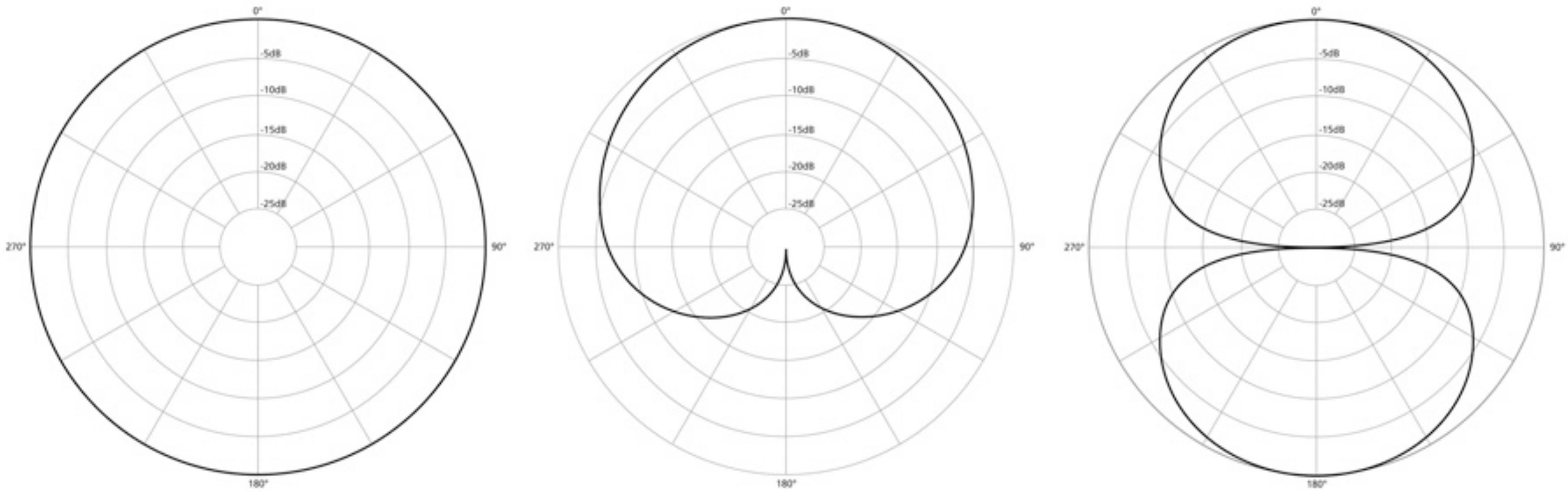
- Hearing aids use electret or MDM mics
  - Electret mics cheaper but susceptible to moisture and temperature changes
  - MEMS microphones more resistant to changes in humidity and temperature, so more stable and consistent over time.
- Directional microphones
  - typically cardioid configuration with 3–4 dB enhancement
- multi-microphone arrays:
  - two separate, well-matched omnidirectional microphones, allow users to switch between omni or directional modes
- Digital MMAs – almost all, today
  - real-time signals from two spatially separated omni mics are digitized and sent to spatial processor that combines signals to produce optimal directivity pattern



www.lifesci.sussex.ac.uk/home/Chris_Darwin

Microphone specs

- Commonly used microphone directional patterns



Omnidirectional Unidirectional (Cardioid) Bi-directional (Figure 8)

- dBm: spec used for microphones = decibels re 1 mW: 0 dBm = 1 mW.
- For power only, but due to historical conventions and equipment 1 dBm it is sometimes taken as 1 mW into a 600 ohm load so 0.775 V signal

Microphone specs

- Sensitivity specs:
  - Free-field sensitivity = voltage generated when in a free sound field (no enclosure) at a sound pressure of 1 Pascal (SPL = 94 dB)
  - xx mV per Pascal @ 1 kHz, or yy dBV/Pascal @ 1 kHz
    - dBV = dB for RMS voltage, relative to 1 V
  - Equivalent noise level (ENL) = self-noise of the microphone
    - Active microphones have built-in circuits that produce self-noise when powered; not present in passive mic types
    - Brownian motion and thermal noise also add to a lesser extent.
    - A-weighted RMS: measure approximates the ear's sensitivity, filtering out low-frequency noise
      - Very low ENL on A scale is typically below 15 dB(A).
    - The ITU-R (International Telecommunication Union, regulates radio communications) BS.468-4 standard
      - Good for comparing noise in condenser microphones as it shows if the microphone suffers from "popcorn noise" or other crackling noise forms.
      - Very low ENL on this scale is typically below 25-30 dB.

www.dpamicrophones.com/

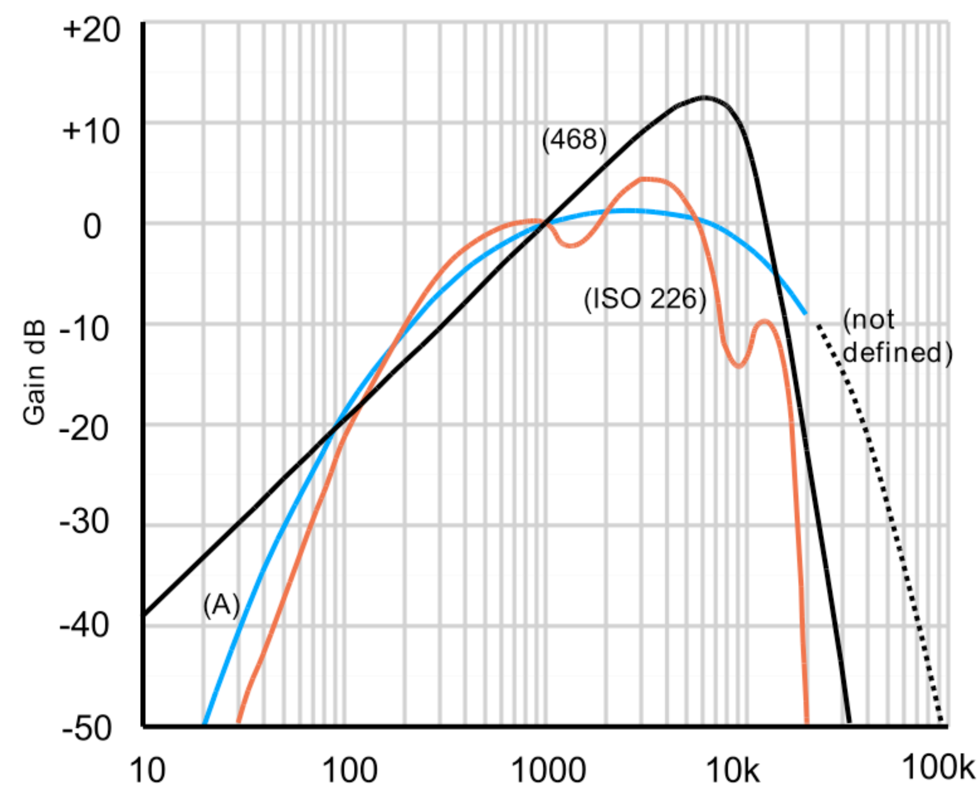
Weightings for mic ENF specs

TU-R 468 response curve (formerly known as CCIR curve), compared to A-weighting, and ISO 226 40-phon equal-loudness contours

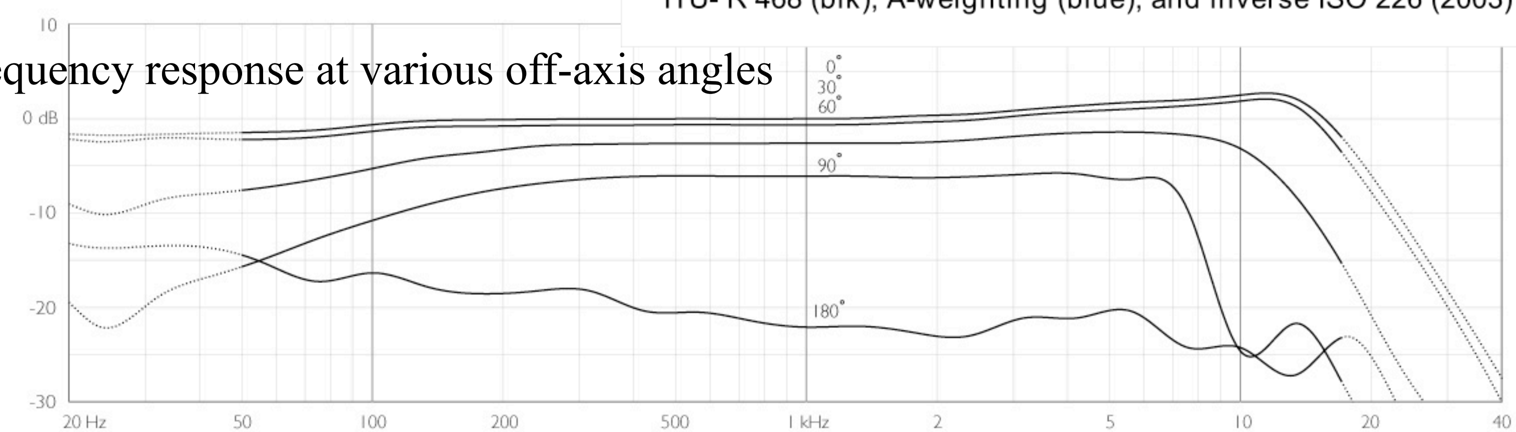
- Example of specs: DPA model 2011 Cardioid Microphone
  - free-field sensitivity, nominal, ±2 dB: 10 mV/Pa; -40 dB re. 1 V/Pa

  (Nominal ±2 dB means: output within 7.9 mV and 12.6 mV exposed 94 dB SPL)

  - Equivalent noise level, A-weighted: 20 dB(A) re. 20 μPa (max. 23 dB(A)) .



ITU- R 468 (blk), A-weighting (blue), and inverse ISO 226 (2003) (red)



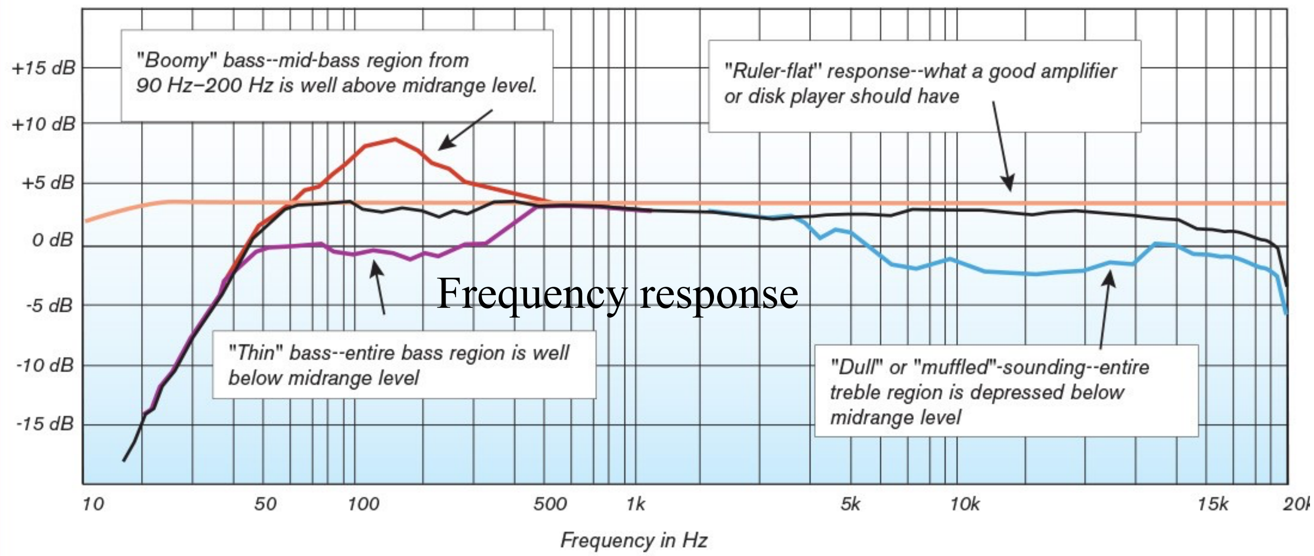
Frequency response at various off-axis angles

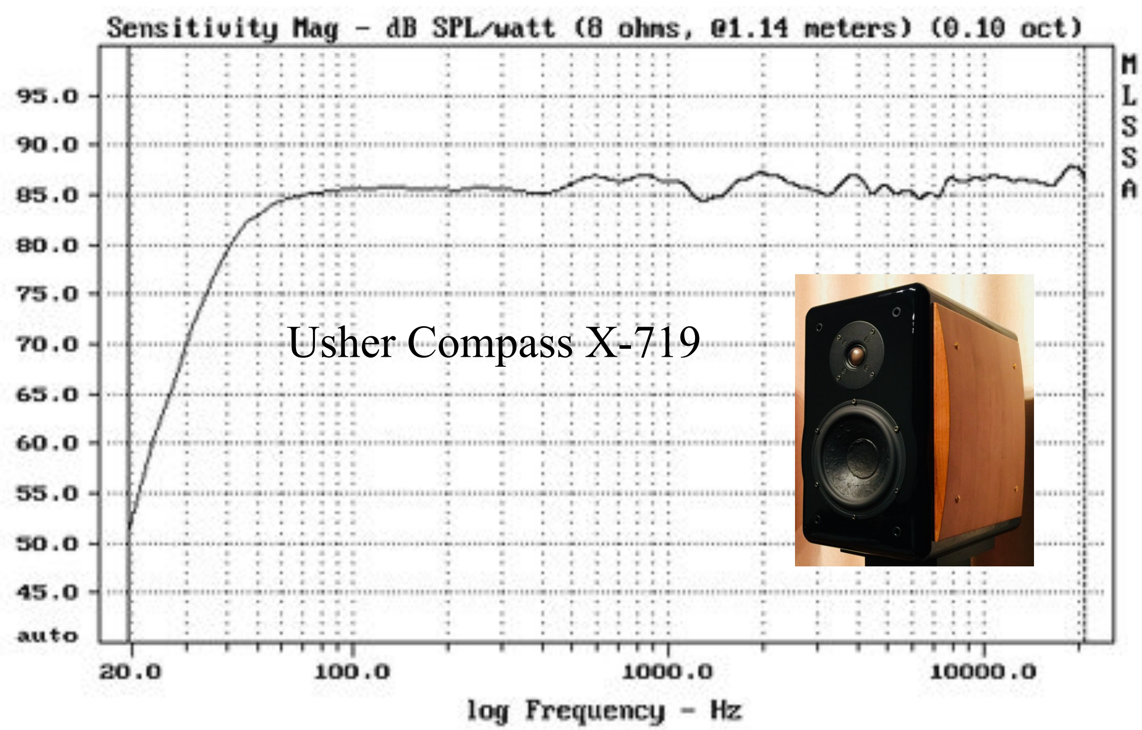
www.dpamicrophones.com/

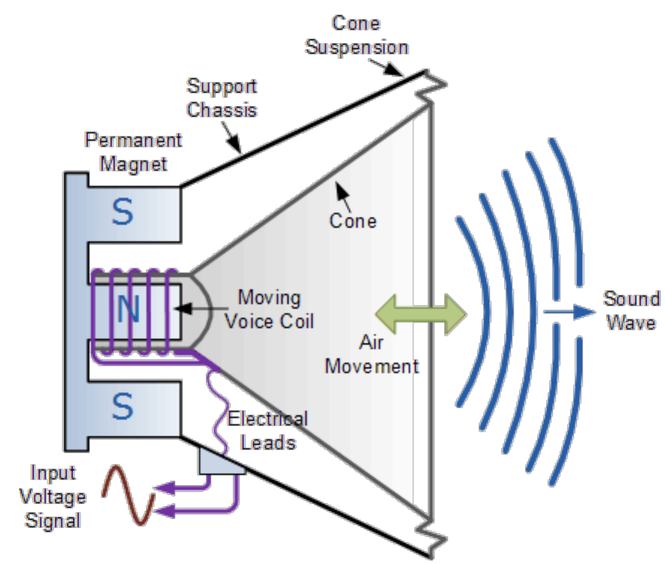
Loudspeakers

Loudspeakers/earphones = dynamic mic in reverse

- Sensitivity=SPL produced at a distance of one meter when 1 W of power is supplied from an amplifier





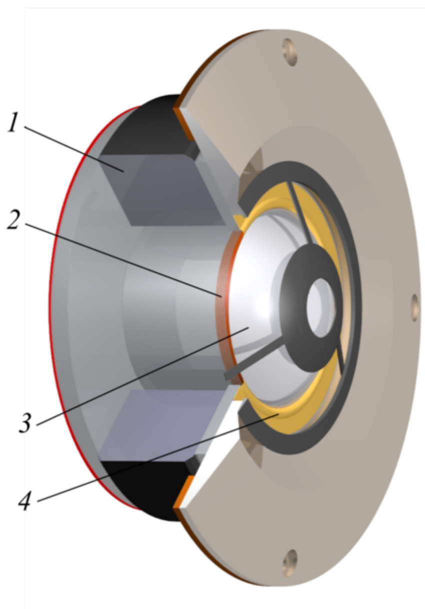


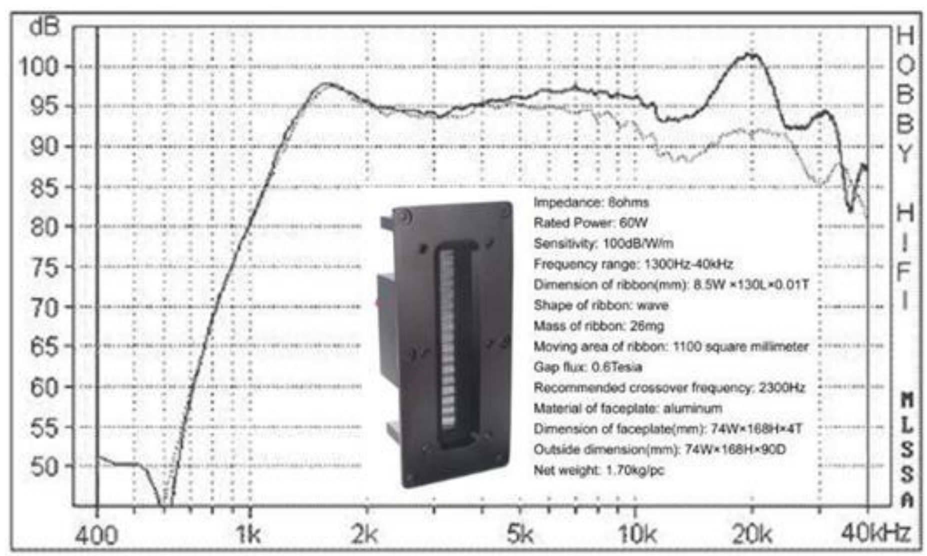
Loudspeakers

- Non-dynamic loudspeakers
  - Most large loudspeakers have electromagnetic drivers
  - Tweeters (high frequency range) may have other types
    - Dome tweeters have dynamic drivers, but voice coil is connected to a dome via a low compliance suspension.
    - Piezo tweeters: have low fidelity; often used in toys, buzzers, alarms
    - ribbon tweeters have useful directional properties: very wide horizontal coverage and very tight vertically

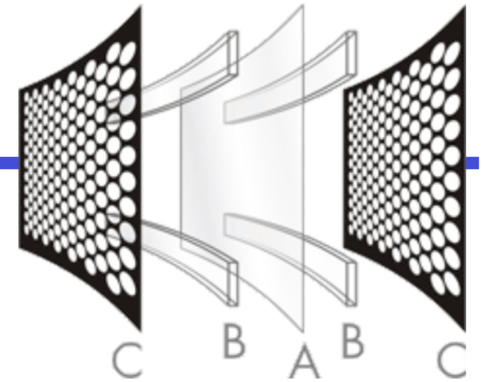
Tweeter with acoustic lens dome membrane

1. Magnet
2. Voicecoil
3. Membrane
4. Suspension



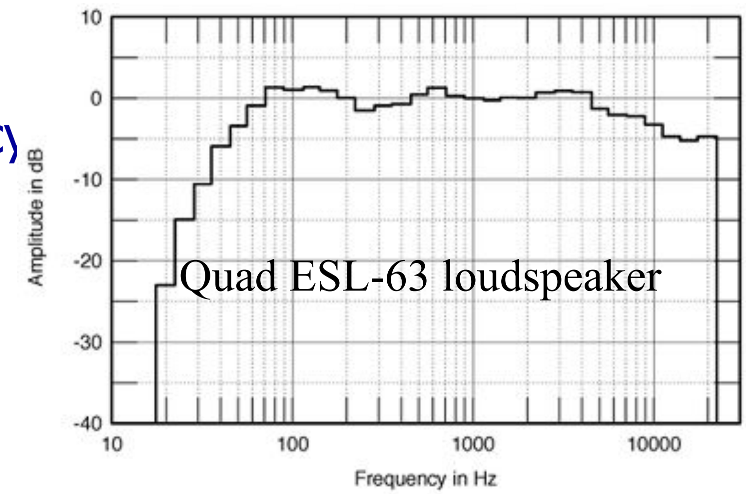


Non-dynamic Loudspeakers



A) Diaphragm
B) Spars
C) Stators
martinlogan.com

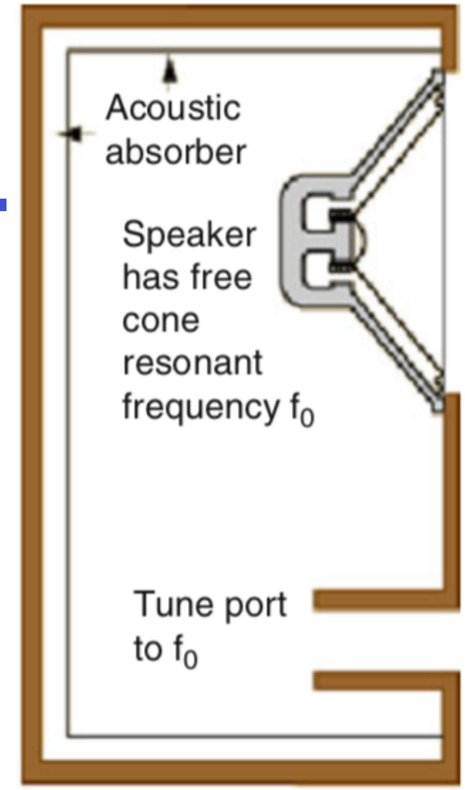
- Electrostatic tweeter: thin diaphragm with conductive coating, suspended between two screens or perforated metal sheets (stators)
  - Old idea (GE 1934; A.Janszen, 1957), but not common due to high cost, low power efficiency, poor bass, fragility
  - Distortion one to two orders of magnitude lower than dynamic drivers
  - Excellent high frequency response: driver is almost free of resonances
  - Electrostatics reduce harmonic distortion* because of their push-pull design. They also have minimal phase distortion.



* Nonlinearities cause distortion, in the form of new harmonics when a pure sine wave fed to the system

Total Harmonic Distortion is ratio of total power in harmonics to power in fundamental: $THD(f) = \sqrt{P_2^2 + P_3^2 + P_4^2 + \ldots} \Big/ P_1$

Bass-Reflex Loudspeaker Enclosure



- Electrodynamic loudspeaker is very inefficient at low frdquencies unless enclosed.
  - Simple enclosure allows only the front surface of the speaker's cone to radiate - half the sound power is wasted
- Helmholtz resonator is driven above its resonance frequency, ωo, produces phase reversal
  - Helmholtz enclosure uses the energy produced by the back of the loudspeaker
- Bass-reflex loudspeaker enclosure* takes the volume velocity generated from the rear of the loudspeaker cone, and inverts its phase
  - At low f, the distance between cone's center and vent (neck of the Helmholtz resonator) is << $\lambda/2$ → motion of air oscillating in the port adds ~ in-phase to motiion driven by the front of the loudspeaker cone.

* L. L. Beranek, Acoustics (McGraw-Hill, 1954)

Bass-Reflex Loudspeaker Enclosure

At frequencies above 33 Hz, the net volume velocity (black solid line) exceeds the volume velocity from the front of the loudspeaker alone (dashed black line), demonstrating the enhancement provided

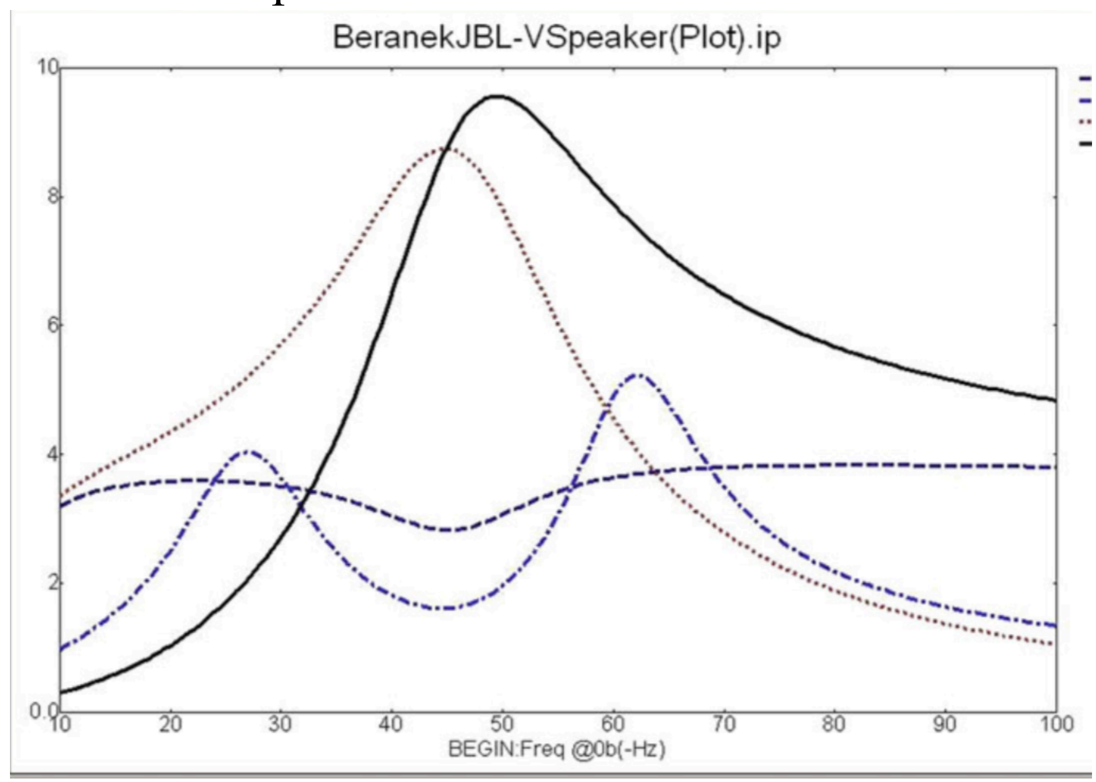
→ Today, very rare to see small loudspeaker enclosures for low-frequency output that are not bass-reflex (Helmholtz resonator)

- - - speaker's input impedance

··· velocity of the air oscillating in the port

- - - velocity produced by the loudspeaker cone

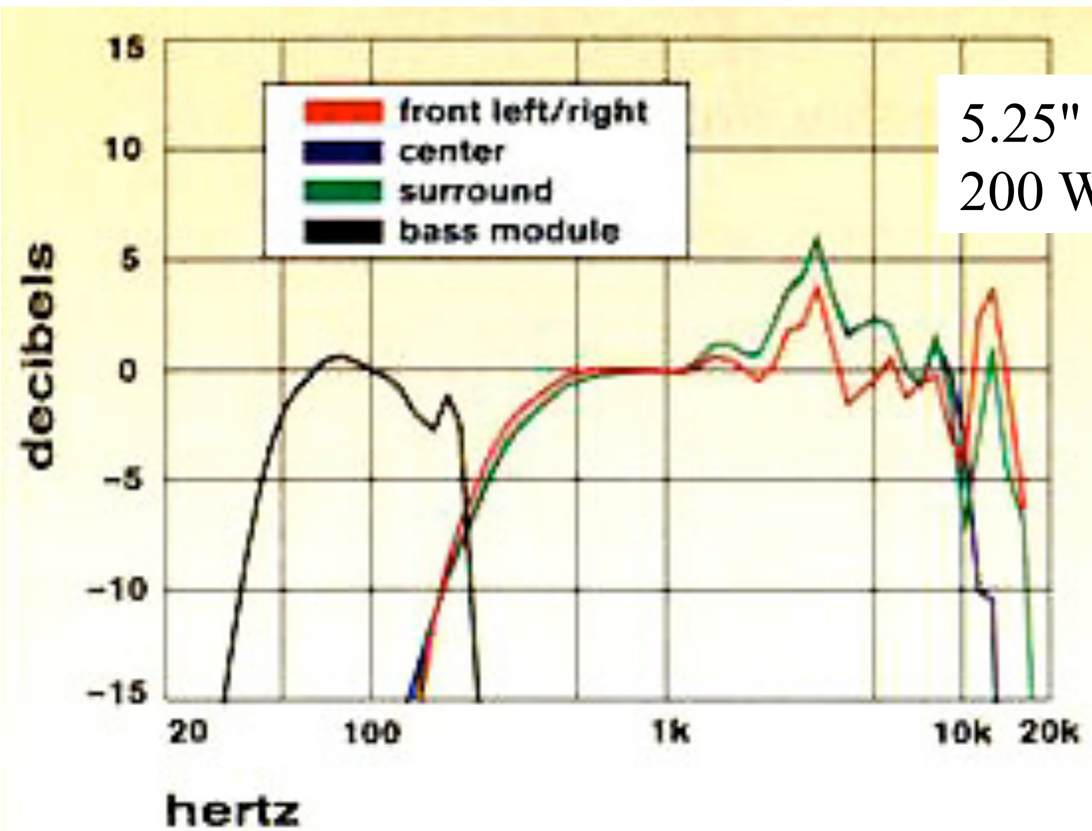
Solid black: magnitude of vector sum of the port and loudspeaker contributions



Example of commercial speakers for audio

- Bose FreeSpace 3 Series / Acoustimass bass module

Frequency Response (+/-3 dB)
210 Hz - 16 kHz / 50 Hz - 230 Hz
Nominal Dispersion 170° conical, Omni-directional below 200 Hz



5.25" (133mm) woofer
200 W (800 W peak)

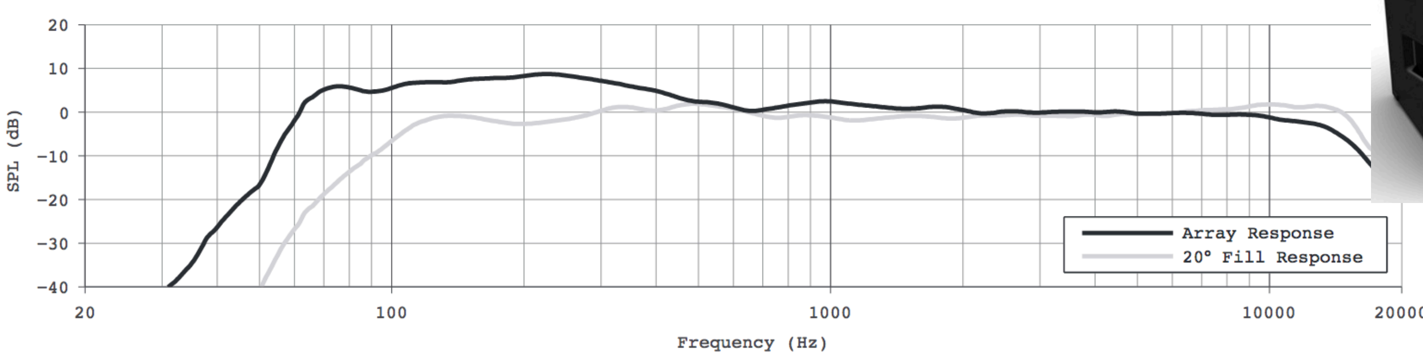
Outdoor high-volume speaker arrays

- Example: Bose SM20 Array

“Requires external power amplifiers and DSP to provide full-range response from 59 – 18,000 Hz with peak array output of up to 145 dB”

Frequency Response (+ / -3 dB) (1) 69 – 16,000 Hz
Frequency Range (-10 dB) 59 – 18,000 Hz
Recommended High-Pass Protection Filter 70 Hz Nominal Coverage Pattern 100° H x 20° V (includes waveguides for 70° H x 20° V)
Recommended Crossover 750 Hz (acoustic; requires active, 2-way crossover in DSP)

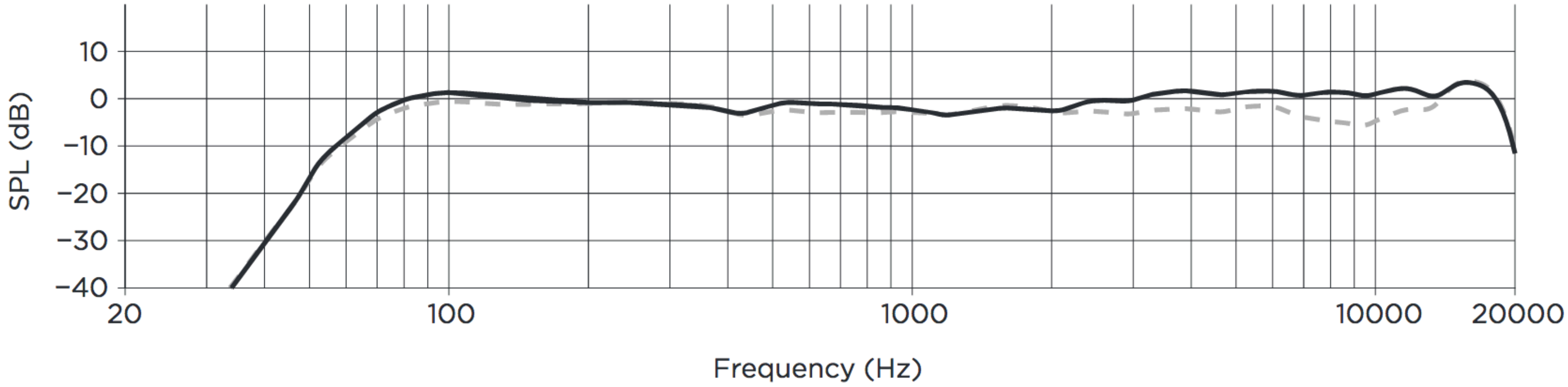
Power Handling, long-term continuous: 450 W (lf) 100 W (hf)
Power Handling, peak: 1800 W (lf) 400 W (hf)



Outdoor and large-volume speakers

- Point-source loudspeakers for sound reinforcement, zone fills, or delays

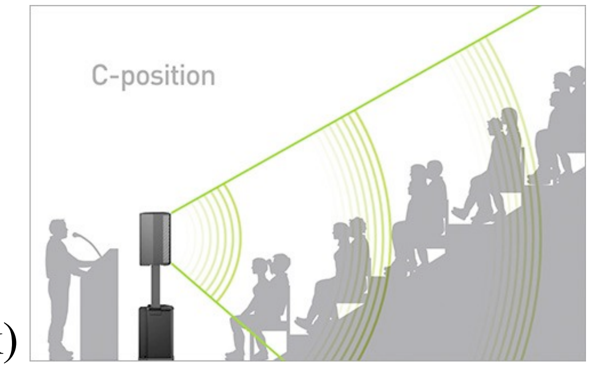
Example: Bose AMM112
12-in (305 mm) woofer
High Frequency 3-in (76 mm) compression driver
Coverage Pattern: 110° horizontal × 60° vertical
Tilt-back Angle 35°
Crossover: Internal passive or external bi-amp
Power Handling, Long-term Continuous 300 W, Peak 2800 W

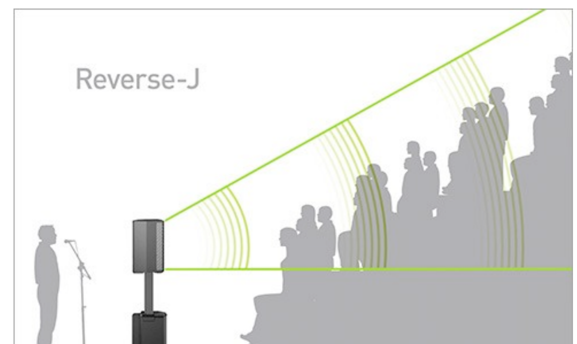


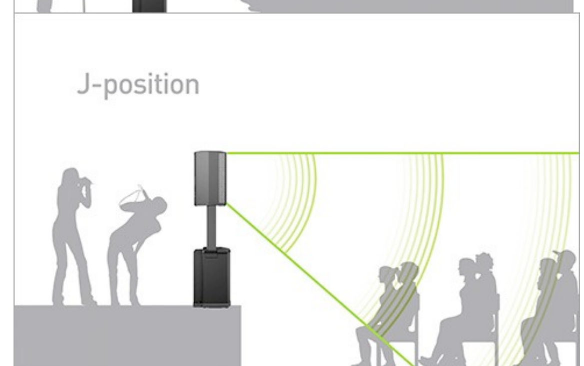
Example: performance-space speakers

- Bose® F1 Model 812 Flexible Array Loudspeaker
- control its vertical coverage pattern - pull the array into position to create pattern wanted, system sets EQ to maintain optimum tonal balance
- Built in 1 kW amplifiers with controls

Dispersion: 100° H x 40° V (C-position)
Maximum SPL @ 1 m: 126 dB SPL (132 dB SPL peak)
Crossover Frequency: 600 Hz 4th order Butterworth



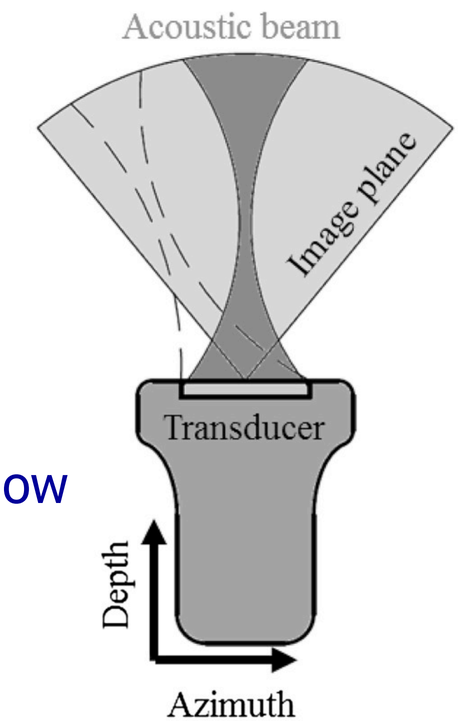


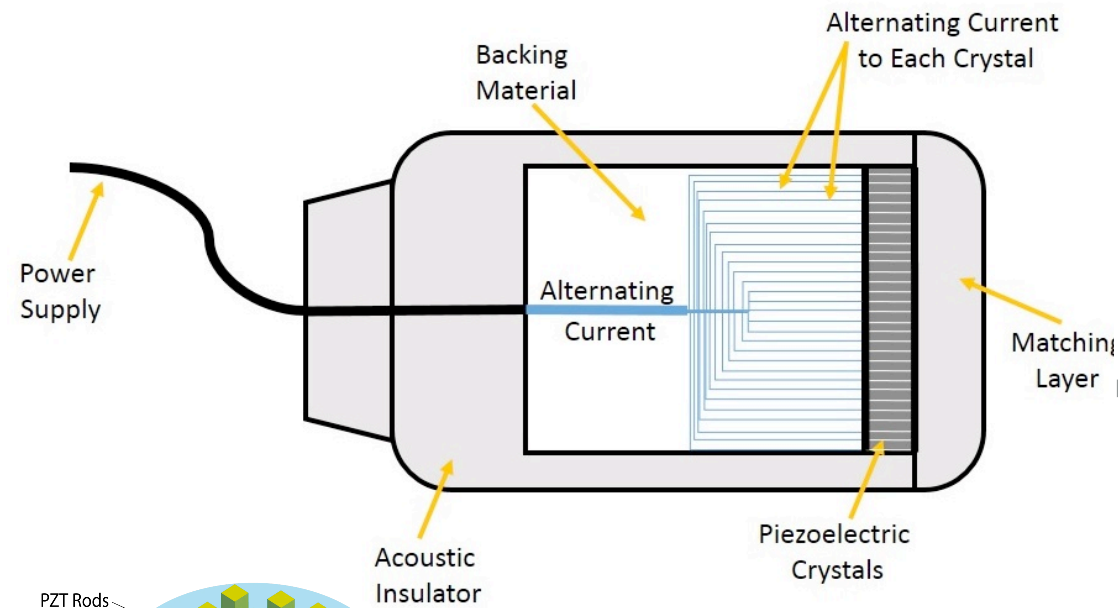


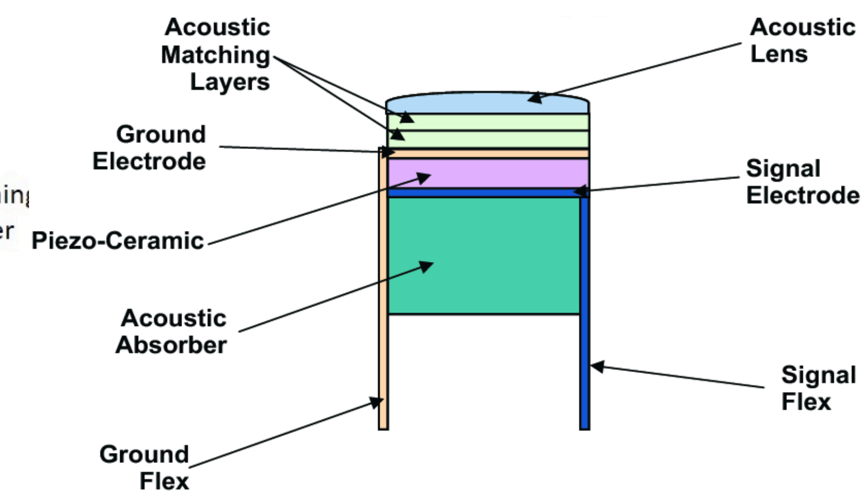
8 vertically mounted drivers, 12-inch LF driver, 2-channel integrated mixer

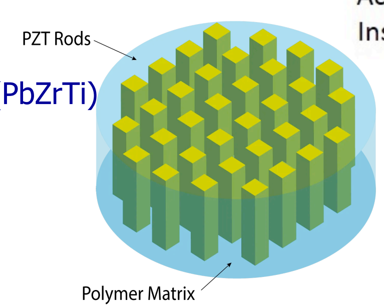
Transducers for ultrasound

- Typically piezos in a linear or 2D array
- Used to form broad beam with neck at suitable radius for high resolution
  - Phased Array transducer has a small footprint and low frequency (its central frequency is 2Mhz – 7.5Mhz).









(PbZrTi)

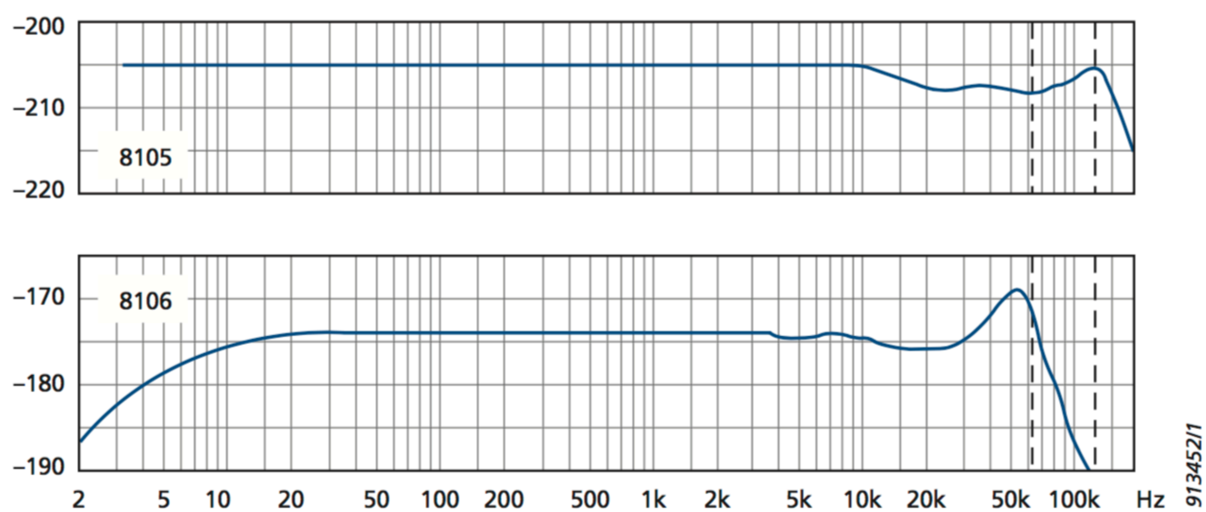
Underwater transducers

- Underwater transducers: hydrophones and projectors ("pingers")
  - Piezo elements are usually ceramic to withstand high pressures
  - Same device may be used as hydrophone or pinger
    - Applying an electrical signal to the transducer produces mechanical vibration in the piezoelectric element.
  - Most devices are rated only for depths of 1~2000m, special types needed for "full ocean depth" (avg 5000m)

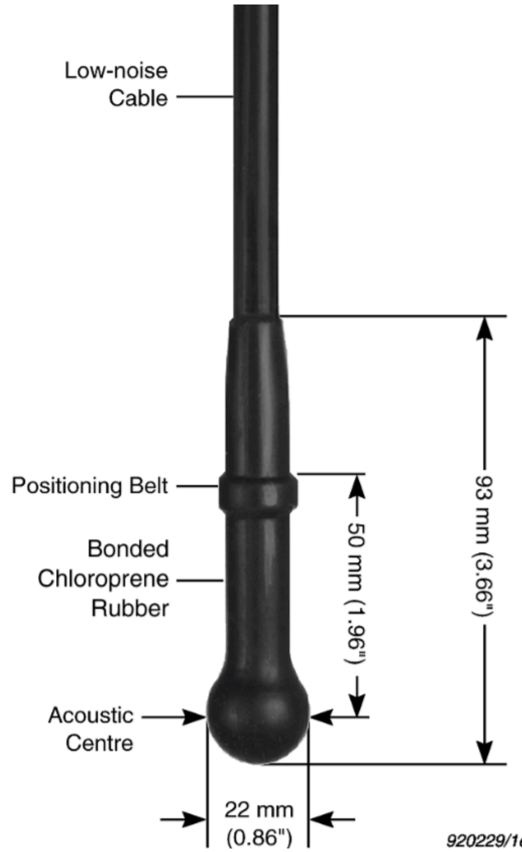
B&K 8105

deep-water sensor in pressure-balanced housing (sea.co.uk)

- Frequency response for:
  - Ominidirectional hydrophone:
  - Low noise hydrophone:



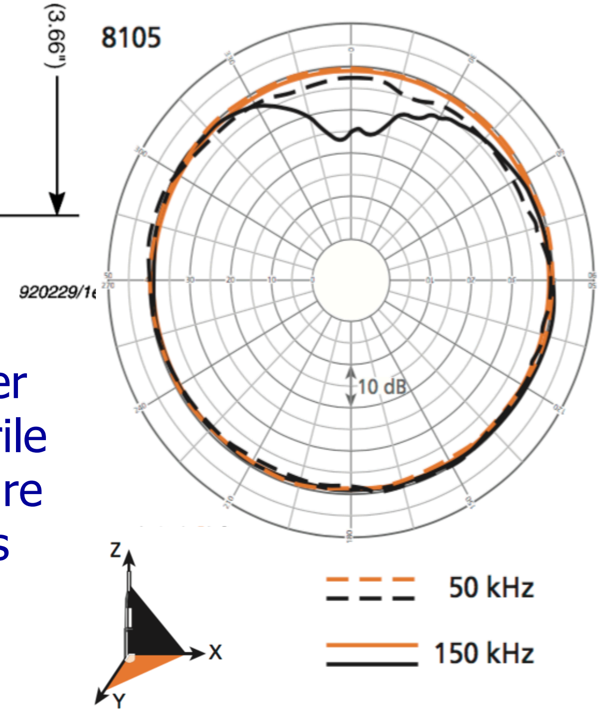
Typical underwater transducer

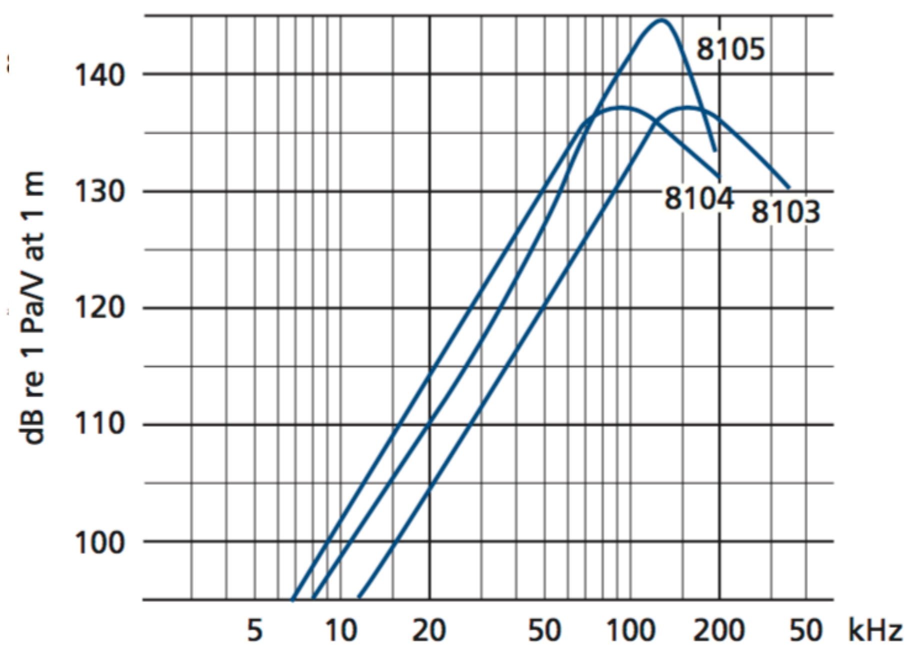


L: Directional sensitivity of omnidirectional hydrophone

R: Transmitting response vs frequency, when used as a pinger

- The 12 dB/octave slope is typical for piezoelectric hydrophones driven with constant RMS voltage input.





Chloroprene rubber (Neoprene) or nitrile rubber (Buna-R) are commonly used as waterproofing for hydrophones

Announcements (repeated)

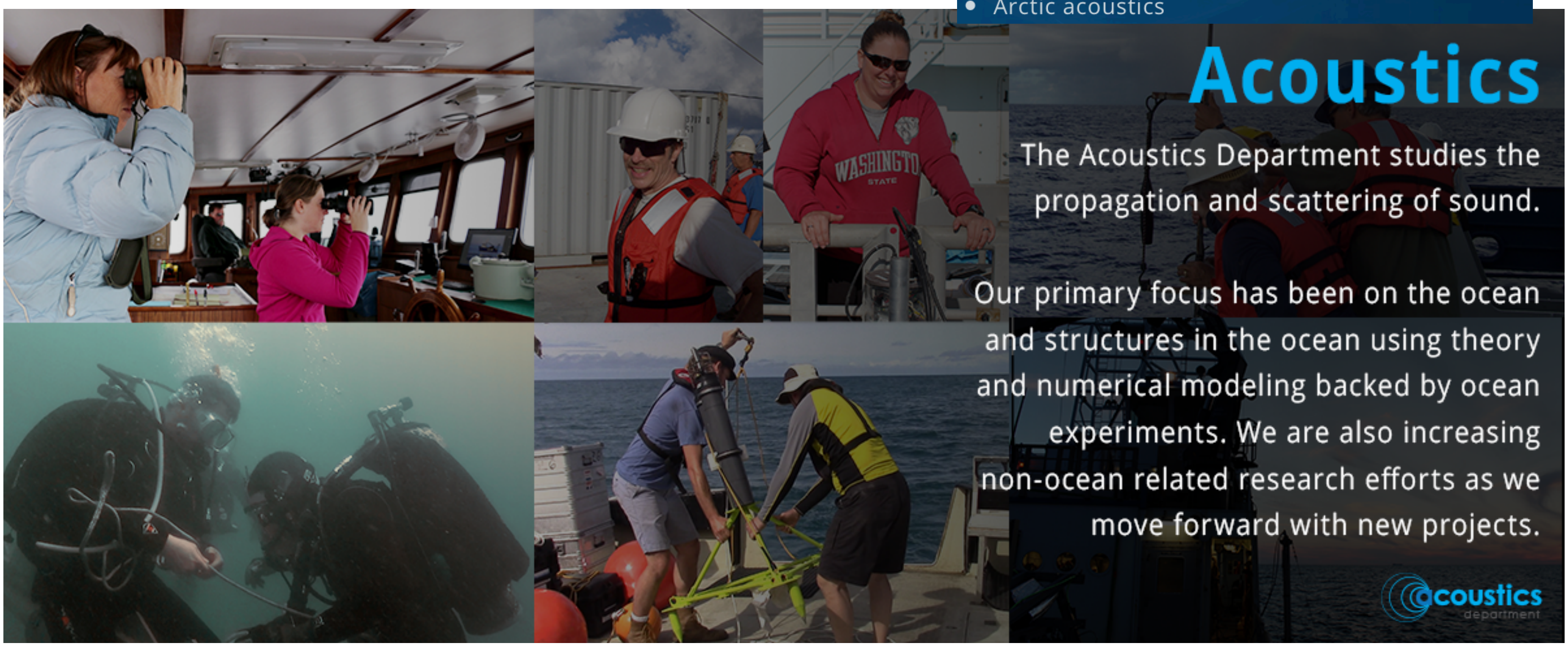
End of term is creeping up – paper 2 is due **in one week**

- Everyone gives a brief (15 min max) presentation in class describing what you learned
  - You should submit your slides via Canvas assignments page on 2/28
    - You can tweak your slides before your presentation time, but for fairness your Canvas submission should be a complete draft
- We need to assign time slots over the 4 nights of presentations
  - Willing to go on the first night (2/28)? pls tell me by email ASAP!
    - Also tell me ASAP if you need a particular day/time for any reason
    - I'll have to arbitrarily assign all remaining slots (random order)
- On the night of your presentation slot
  - **You must email me your final slides as a pdf or ppt file**
    - Slides and notes must be sent by **6 pm on night you present**
  - I will share your screen – but if you do not have adequate setup, let me know and I will display your sides
  - You MUST have a microphone – please test your audio before class

Guest Speaker Thursday

Bob Odom, Sr. Principal Physicist, UW Applied Physics Lab (see www.apl.washington.edu), Prof. Emeritus of Earth Sciences

- Acoustic propagation and scattering from volume heterogeneities and surface roughness
- Undersea acoustic communications
- Remote sensing of the environment
- Acoustical oceanography
- Scattering from objects deployed in the ocean
- Ambient noise in air and underwater environments
- Measurement and control of underwater industrial noise
- Arctic acoustics



Underwater acoustics

- Propagation of sound in the ocean is of great interest to many mammals (and other classes of animals)
  - We'll focus on acoustics of seawater
  - Jumping ahead for a quick intro before our guest speakers
- Speed of sound in water
  - Freshwater c depends on T and P
  - Seawater adds another factor: salinity
  - Formulas for c
    - Historic example: Wilson formula (JASA 32:641, 1960): coefficients for a polynomial fit with standard deviation 0.22 m/sec.
    - Simplified formula (Medwin, JASA 58:1318, 1975)
      - T = deg C, z=depth in m, S=salinity in ppt (0/00)

$$c = 1449.2 + 4.6T - 0.055T^2 + 0.00029T^3 + (1.34 - 0.010T)(S - 35) + 0.016z$$

Underwater acoustics

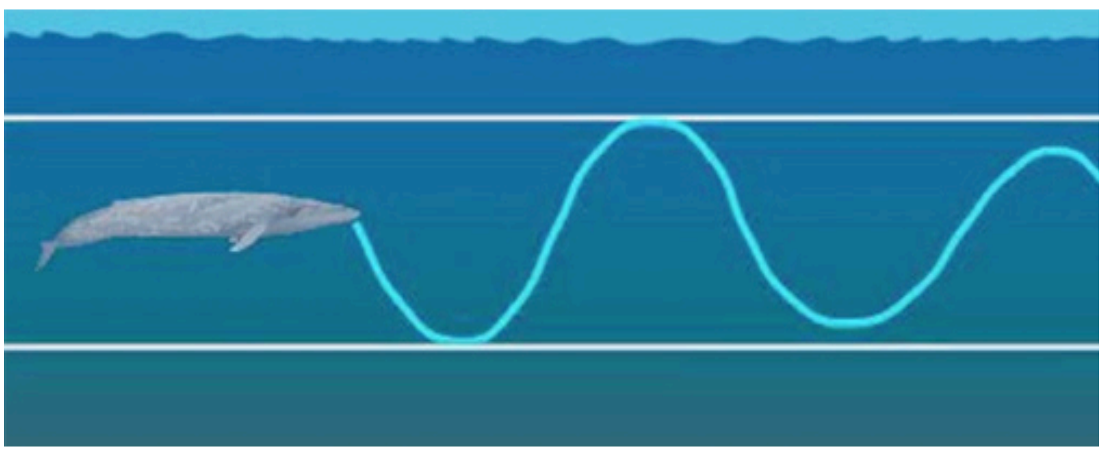
- Famous feature of ocean sound speed vs depth:
  - Reversing slope at ~1 km

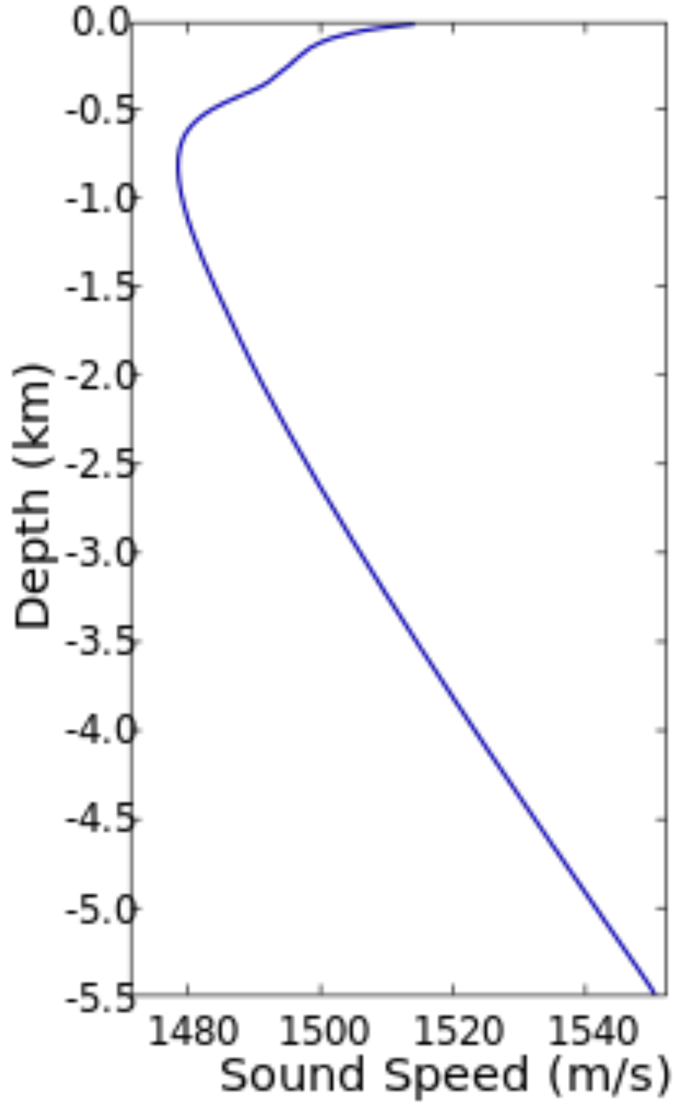
Recall: sound rays refract when c changes

  - Produces a trap for sound rays:

Deep Sound Channel

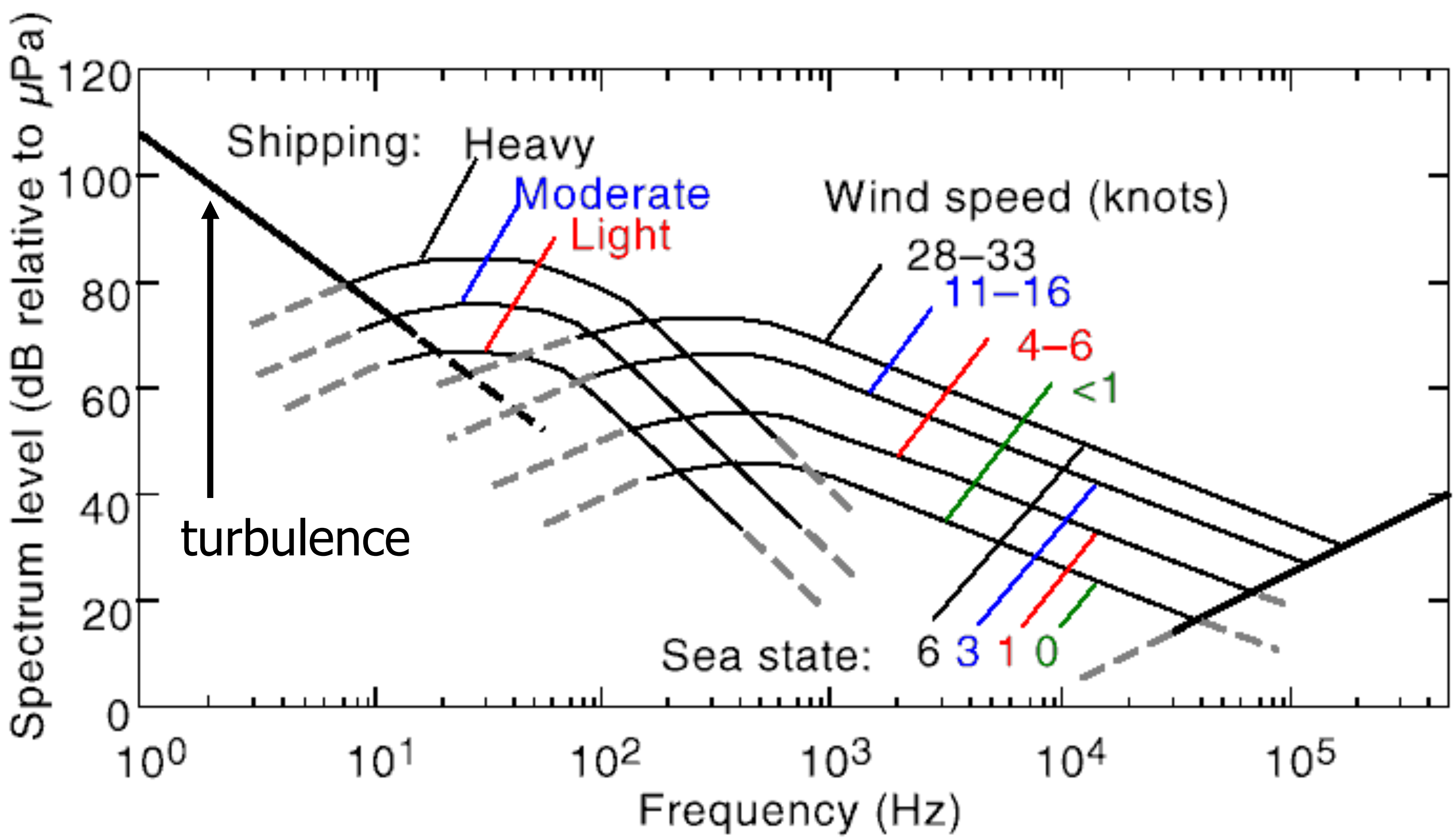
  - Acts like a waveguide for sound
  - Sound can travel great distances
  - Surface observers can't hear sounds produced in the channel





Underwater acoustics

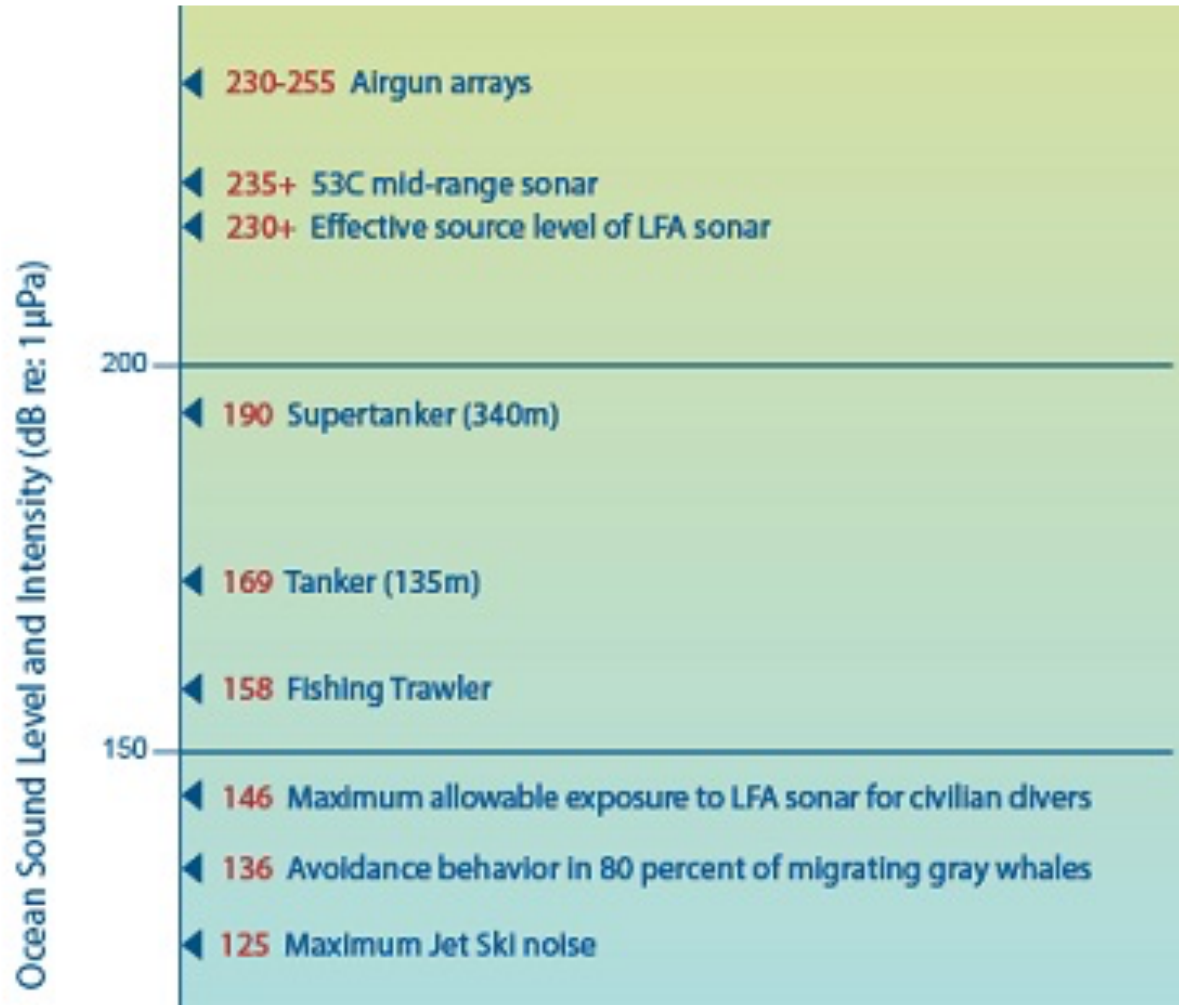
- Background noise underwater
  - Shipping noise has increased greatly since 1983!



Average deep-water ambient noise spectra

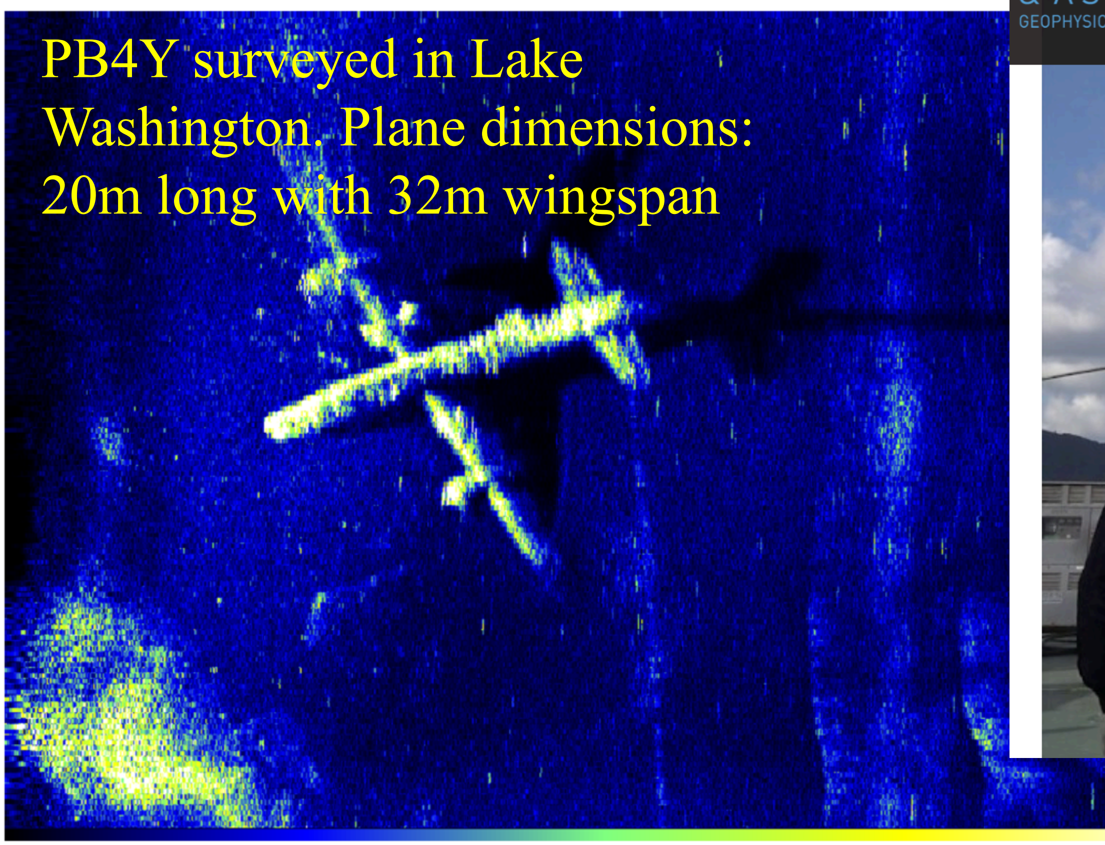
Underwater acoustics

- Major noise pollution sources
  - Broadband dB re 1 microPa



Speakers next week

Tues: Art Wright, VP for Operations, Williamson Associates Ocean Engineering (see www.wassoc.com)



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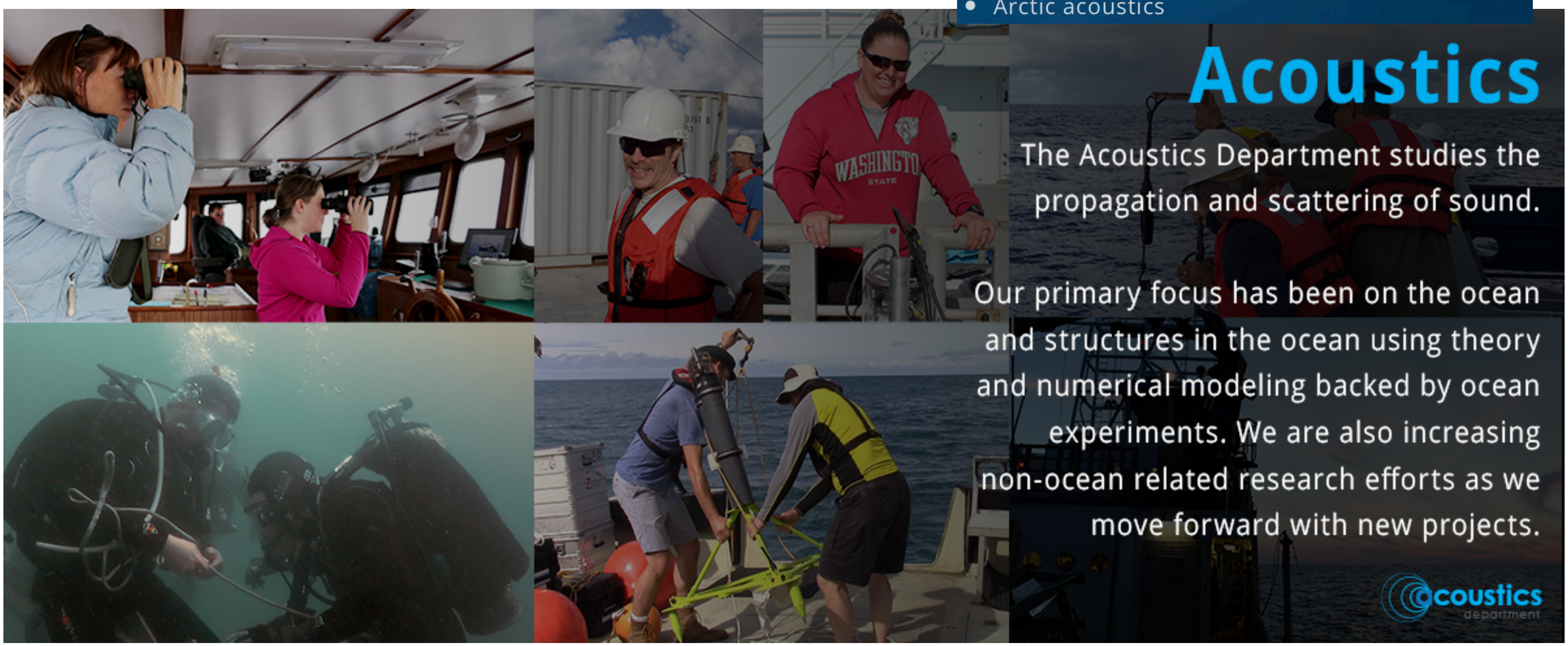
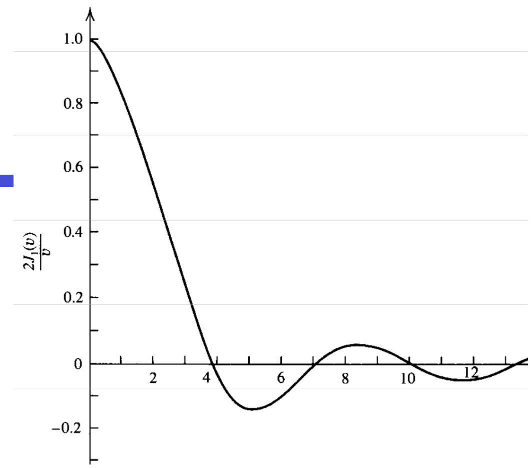


https://www.mne.psu.edu/
lamancusa/me458/

Far field for plane circular piston source



Integral → Bessel function: $\int_0^{\pi} \cos[z\cos\phi]\sin^2\phi\, d\phi = \pi J_1(z)/z$

so $p(r,\theta,t) = \frac{i}{2}\rho_0 c U_0 \left(\frac{a}{r}\right) ka \left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right] e^{i(\omega t - kr)}$

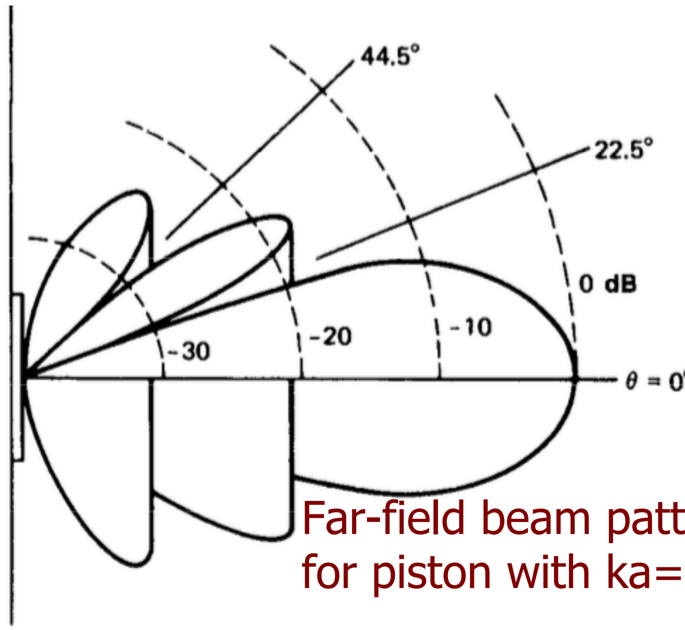
- All the angular dependence is in the square brackets
  [ ] →1 as θ →0, so we can factorize:

$|p(r,\theta)| = P_{AXIAL}(r)H(\theta)$, where $H(\theta) = \left|\frac{2J_1(v)}{v}\right|$, with $v = ka\sin\theta$

$P_{AXIAL}(r) = \frac{i}{2}\rho_0 c U_0 \left(\frac{a}{r}\right) ka$

There is a maximum at $\theta = 0$,
and pressure nodes at angles where $J_1(v) = 0$:
$v_m = ka\sin\theta_m, \quad m = 1,2,3\ldots$

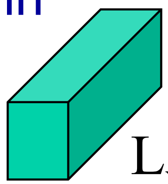
For ka >>1 (λ < a), many narrow lobes;
For ka ~ 1, one broad lobe
For ka << 1 (long λ) → pattern of simple source with strength $Q=\pi a^2 U_0$



Far-field beam pattern for piston with ka=10

Standing waves in rectangular cavities

- Sound boxes for musical instruments and building ducts act as cavity resonators for sound
  - We can apply similar procedure as with pipes, except now in Cartesian coordinates and with no openings
    - Cavity has dimensions L_x , L_y , L_z,
    - Assume walls are rigid (particle speed u=0 at wall)
    - Repeat procedure used for 2D case in membranes:



$$\nabla^2 p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \quad \rightarrow \quad \left.\frac{\partial p}{\partial x}\right|_{x=0,\,L_x} \left.\frac{\partial p}{\partial y}\right|_{y=0,\,L_y} = \left.\frac{\partial p}{\partial z}\right|_{z=0,\,L_z} = 0 \quad \left(u = 0 \text{ at walls}\right)$$

  - Separation of variables:

$$p = X(x)Y(y)Z(z)\, e^{i\omega t} \rightarrow \left(\frac{d^2}{dx^2} + k_x^2\right)X = 0, \quad \text{same for } y,\ z$$

separate constants must be related: $\omega / c = k^2 = k_x^2 + k_y^2 + k_z^2$

$$u = 0 \text{ at walls } \rightarrow p = A_{lmn}\cos\left(k_{xl}x\right)\cos\left(k_{ym}y\right)\cos\left(k_{zn}z\right)\exp\left(i\omega_{lmn}t\right)$$

with $k_{xl} = l\pi / L_x,\ k_{ym} = m\pi / L_y,\ k_{zn} = n\pi / L_z, \quad \{l,m,n\} = 0,1,2...$

Allowed ωs are quantized: $\omega_{lmn} = \sqrt{\left(l\pi / L_x\right)^2 + \left(m\pi / L_y\right)^2 + \left(n\pi / L_z\right)^2}$

Driven longitudinal vibrations and resonance in bars

- Bar (length L) is driven at x=0 by force F_0 exp(iωt), and support at other end x=L has mechanical impedance Z_{mL}
  - At the driven end, $F_0 e^{i\omega t} = -\rho_L c^2 \left.\frac{\partial \zeta}{\partial x}\right|_{x=0}$ with $\rho_L = \rho S, \quad c^2 = \frac{Y}{\rho}$
  - At x=L, boundary condition is

$$Z_{m(L)} = \frac{f_L}{u(L,t)} \quad \rightarrow \quad \left.\left(\frac{\partial \zeta}{\partial x}\right)\right|_{x=L} = -\frac{Z_{m(L)}}{\rho_L c^2}\left.\left(\frac{\partial \zeta}{\partial t}\right)\right|_{x=L}$$

$$\zeta(x,t) = Ae^{i(\omega t - kx)} + Be^{i(\omega t + kx)} \quad \text{with wave number } k = \omega / c$$

(Same as for fixed-end bar)