Digitally Replicating Acoustic Pianos

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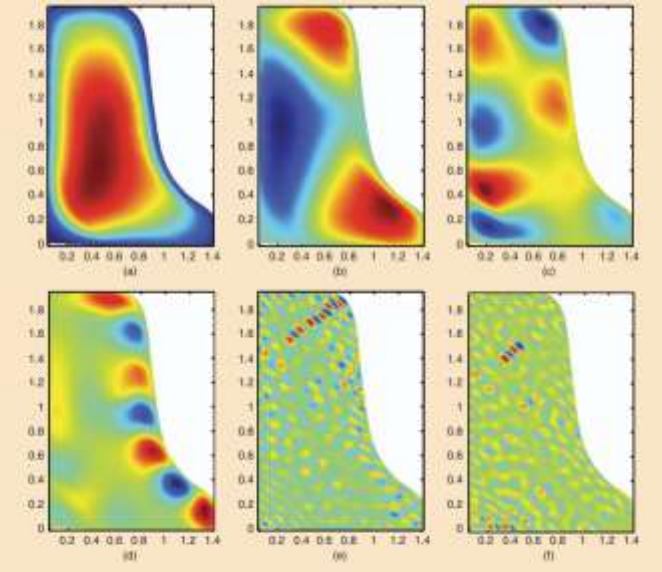
Introduction

- Digital pianos offer a lot of benefits including portability, ability to use headphones, and different voices (instruments).
- Digital pianos need to faithfully reproduce the sound and feel of a concert grand piano
- Replicating that sound within the computational and memory constraints of digital pianos is challenging

Approaches to Digital Reproduction

Acoustic Sampling

Digital Synthesis



Acoustic Sampling

- Record every note at a variety of key velocities ("Velocity Layers")
  - Cheaper pianos sometimes only sample 1 note in a group and then modify its pitch
- Compress audio to minimize file size
- When a key is pressed, play the corresponding sample based on key velocity
  - Long decay is often looped to save memory
- Post processing adds reverberation and other effects

What affects the quality of the sampling?

- Quality of the recording
  - High quality anechoic chamber
  - Well-tuned piano
- Number of velocity layers
  - Hitting the key harder does more than just change the volume
- Compression of audio file
  - Cheaper pianos have less memory and may require lossy compression
- Looping resonant decay
  - Different resonances decay at different rates, so looping is not very accurate
- Advanced features
  - Sympathetic resonance between strings
  - Binaural recording

Why Digital Synthesis?

- Prevents expensive and timely recording sessions
  - Takes months for a recording session
- Reduces memory requirements
  - Does require more powerful processors
- Allows greater flexibility to tweak the sound of the piano
- Increased understanding of the physics behind pianos

Lid − 4 +
0 6

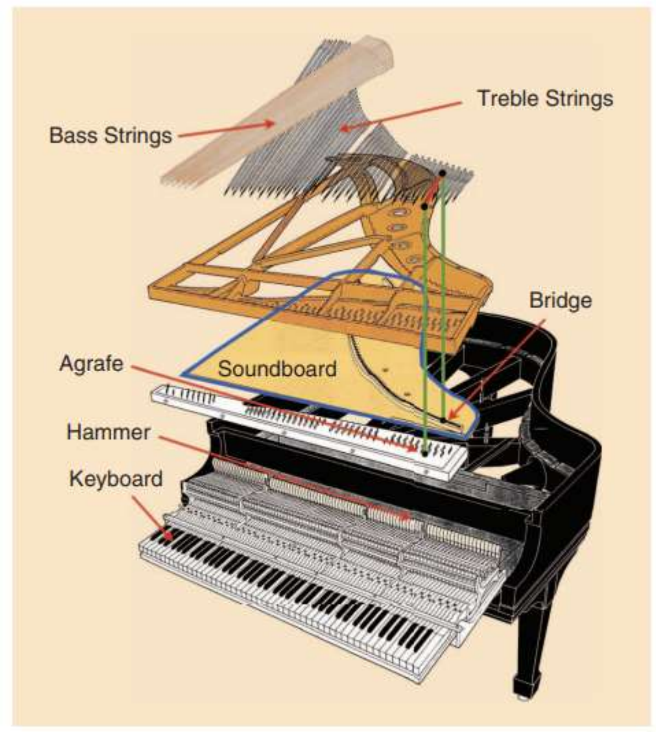
Cabinet Resonance − 3 +
Off 10

Soundboard Type − 2 +
1 5

Hammer Noise − -2 +
-2 2

How a Piano Works

- Key is pressed
- Action lifts the damper and hits the string with the hammer
- String resonates
- Sound is amplified through the sound board



Model of a Piano String

$$\mu \frac{\partial^2 y}{\partial t^2} = c^2 \cdot \frac{\partial^2 y}{\partial x^2}$$

- Start with the wave equation for a string:

Model of a Piano String

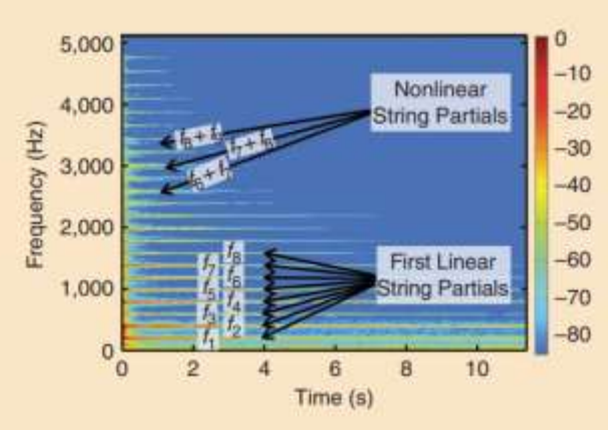
$$\mu \frac{\partial^2 y}{\partial t^2} = c^2 \cdot \frac{\partial^2 y}{\partial x^2} \boxed{- 2R\mu \frac{\partial y}{\partial t} + 2\eta\mu \frac{\partial^3 y}{\partial t\, \partial x^2}}$$

- Add decay factor caused by radiation losses:
    - First term is a constant loss
    - Second term accounts for frequency dependent losses

Model of a Piano String

$$\mu \frac{\partial^2 y}{\partial t^2} = c^2 \cdot \frac{\partial^2 y}{\partial x^2} - 2R\mu \frac{\partial y}{\partial t} + 2\eta\mu \frac{\partial^3 y}{\partial t\, \partial x^2} \boxed{- ES\kappa^2 \frac{\partial y^4}{\partial x^4}}$$

- Piano strings are quite stiff
- Causes higher frequency resonances that decay faster
- Can be compared to vibrational modes of metal bars
- E= Young’s Modulous



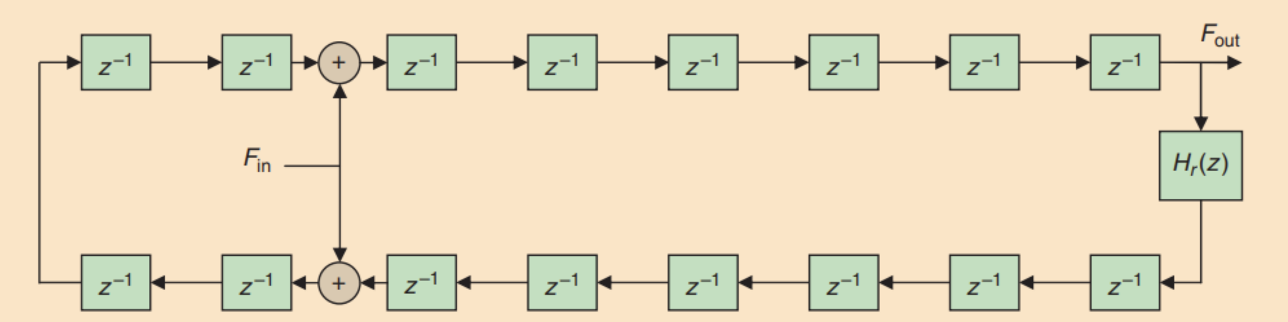
Model of a Piano String

$$\mu \frac{\partial^2 y}{\partial t^2} = c^2 \cdot \frac{\partial^2 y}{\partial x^2} - 2R\mu \frac{\partial y}{\partial t} + 2\eta\mu \frac{\partial^3 y}{\partial t\, \partial x^2} - ES\kappa^2 \frac{\partial y^4}{\partial x^4} \boxed{+ d_y(x, t)}$$

- d_y accounts for the external force of the hammer

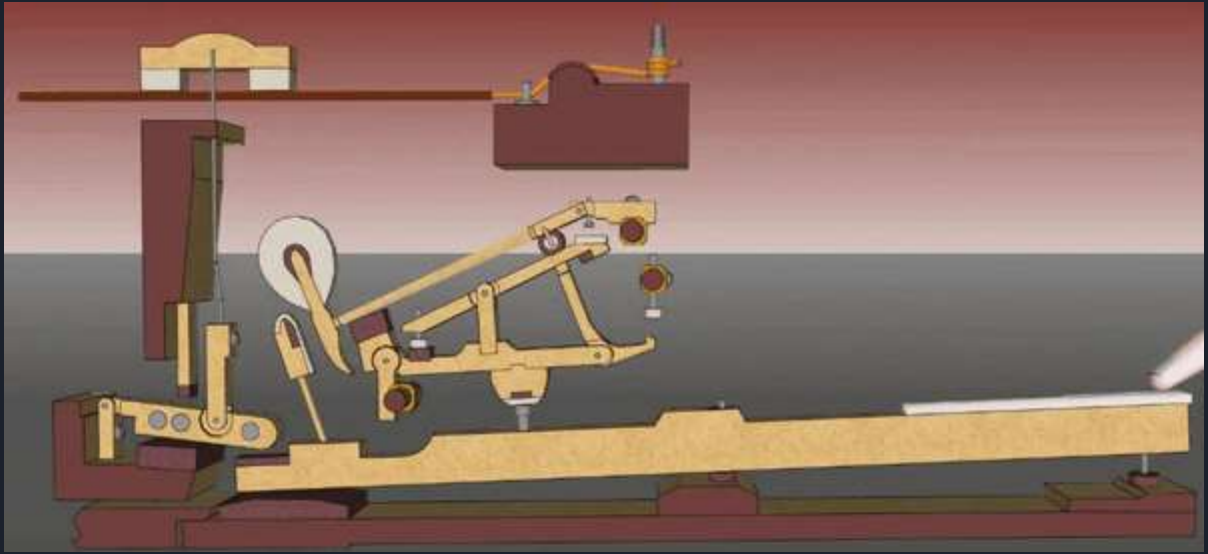
Model of a Piano String

- Differential equation can be processed using Finite Element Modeling (FEM)
- Can also be modelled analytically as a travelling wave
    - F_{in} represents hammer strike
    - F_{out} represents connection to resonating sound board
    - $H_r(z)$ is a “reflection filter”, modelling losses



Model of a Hammer

What is $d_y(x, t)$?



Model of a Hammer

- Simplest model – impulse response
- More detailed model– a small mass connected to a non-linear spring
    - Models the compression of the felt in the hammer

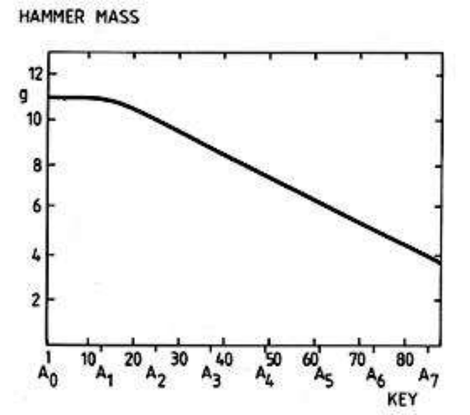
$$F_h(\Delta y) = \begin{cases} K_h \cdot \Delta y^{P_h} & if\ \Delta y > 0 \\ 0 & if\ \Delta y \leq 0 \end{cases}$$

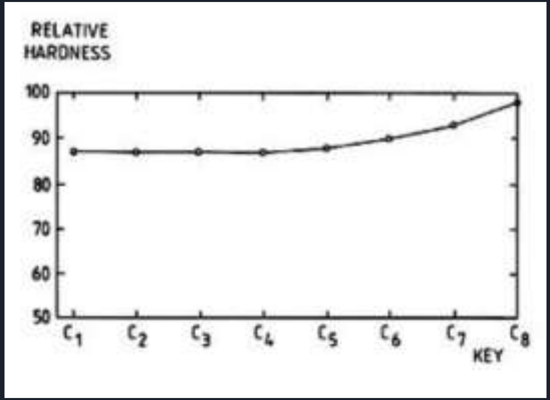
$$F_h(t) = -m_h \frac{d^2 y_h(t)}{dt^2}$$

- $F_h = Force\ of\ the\ hammer\ onto\ the\ string$
- $\Delta y = y_h(t) - y_s(t) = Compression\ of\ the\ hammer\ felt$
- $y_h(t) = Position\ of\ the\ hammer$
- $y_s(t) = Position\ of\ the\ string$
- $K_h = Stiffness\ coefficient$
- $P_h = Stiffness\ exponent$
- $m_h = Mass\ of\ the\ hammer$

Model of a Hammer

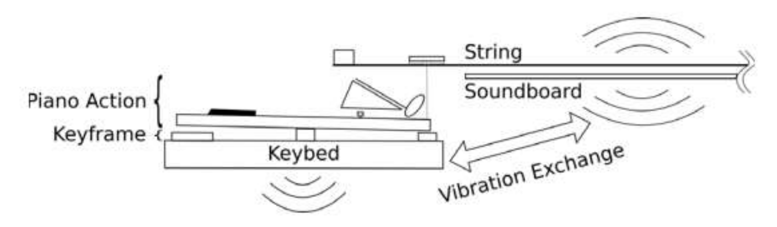
- More complex models:
    - The wooden linkage resonates around 260Hz
    - Hammer mass and felt durometer changes depending on the note
    - Dampers have a complex effect in dissipating the sound

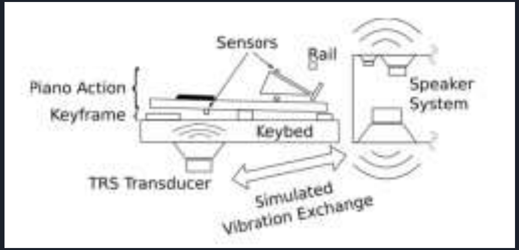




Model of a Hammer

- The key action vibrates the key, giving tactile feedback
- Key vibrates below 500Hz
- Fingertip is most sensitive to vibrations between 200 to 300Hz
- High-end digital pianos simulate this tactile feedback





Soundboard

- Soundboards are complex resonators to amplify the sound from the string
- Most computationally intensive part to model
- Can be modelled using Kirchoff-Love Equations for 2D plates using FEM

Fig. 17. First (lowest) soundboard mode at 49 Hz.

Fig. 18. Second mode at 67 Hz.

Fig. 19. Third mode at 89 Hz.

Fig. 20. Eighth mode at 184 Hz.

Soundboard

- Commuted Synthesis used to simplify computation
  - Assume a linear time-invariant (LTI) system
  - Sound board can be modelled as a finite impulse response (FIR) filter
  - Prevents modelling of non-linearities such as restrike
  - Computationally much simpler

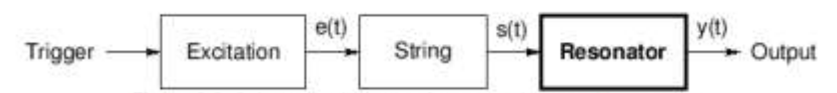


Figure 8.9: Schematic diagram of a stringed musical instrument.

Figure 8.10: Equivalent diagram in the linear, time-invariant case.

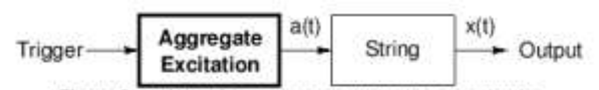
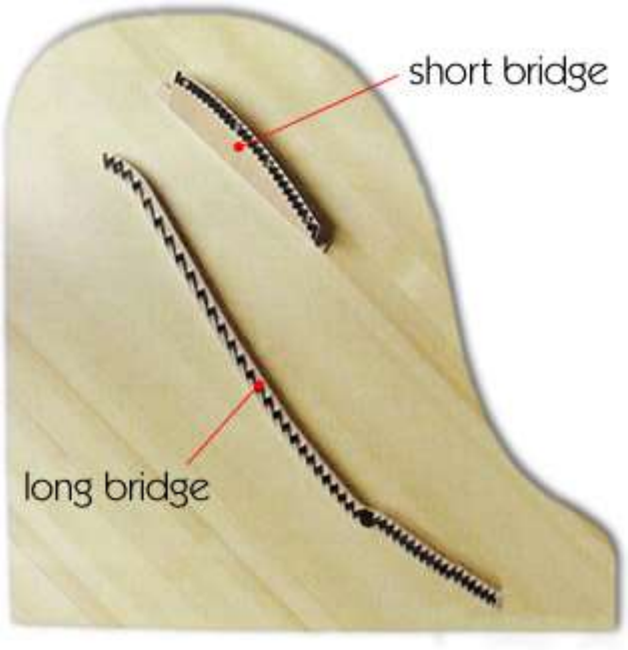


Figure 8.11: Use of an aggregate excitation given by the convolution of original excitation with the resonator impulse response.

Bridge

- The bridge couples the string vibrations to the soundboard
- Most models assume ideal coupling
- Some more advanced models take non-idealities into account



Conclusion

- Digitally recreating a 300-year-old instrument is challenging
- Acoustic sampling is possible but is expensive and inflexible
- String resonance is more complicated than the ideal wave equation
- Hammers strike the string in a complex way
- Sound boards provide important amplification but are hard to model
- Trade-off in accuracy vs. computational needs

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