Topological Acoustics with Applications in Waveguides

PHYS536

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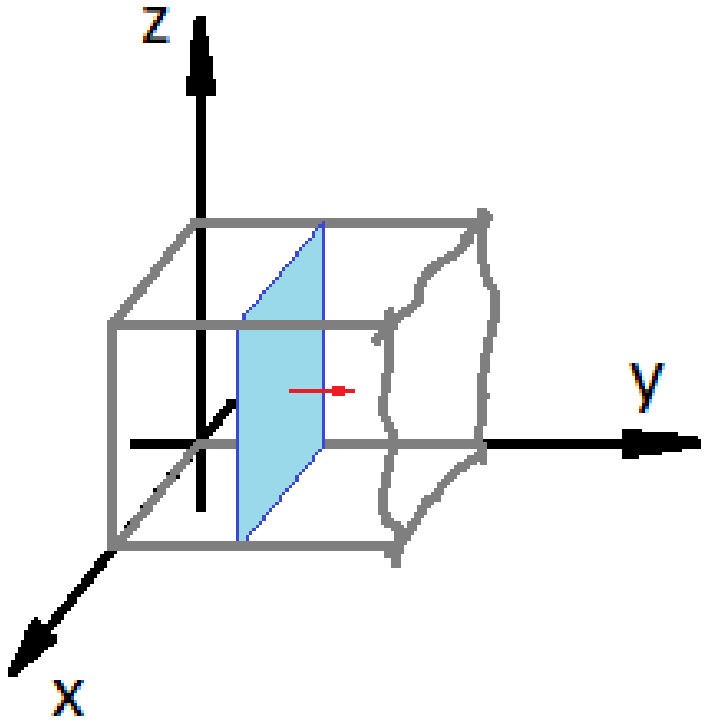
Waveguides

- Guides Wave along Path
  - Similarities with Cavity Solution
  - Free Boundary Permits Wave Propagation

- Ref. [1] Example, Rectangular Waveguide

$$p_{lm} = A_{lm} \cos(k_{zl} z) \cos(k_{xm} x) e^{j(\omega t - k_y y)}$$

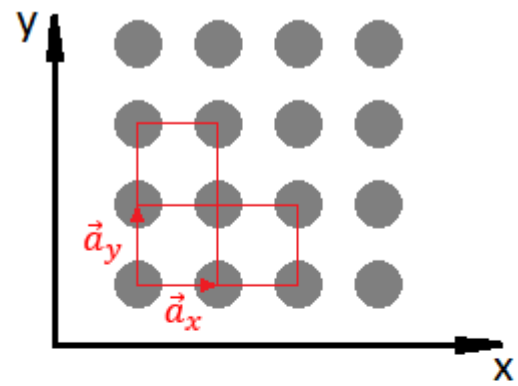
Traveling Wave with
Continuous Wave Number

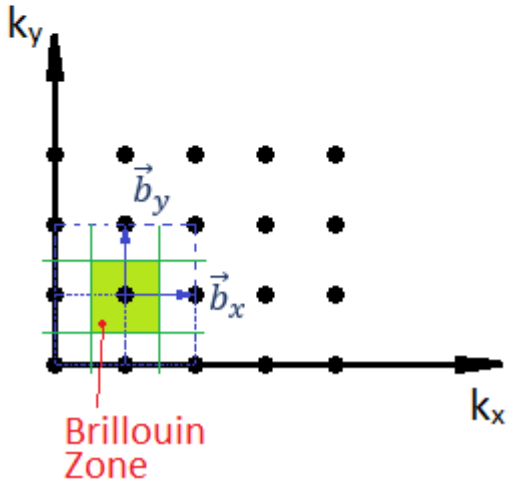


Crystals Overview

- Sonic & Phononic Crystals
  - Composite Solids
    - Solid Compression and Transverse Waves
  - Fluid Acoustic Waves
    - Resonator Lattice
    - Solid Lattice
- Reciprocal Lattice
  - $\vec{a}_i \cdot \vec{b}_i = 2\pi\delta_{ij}$
  - Lattice Vectors

    $\vec{R} = \sum_i n_i \vec{a}_i \qquad \vec{G} = \sum_i n_i \vec{b}_i$
  - Brillouin Zone





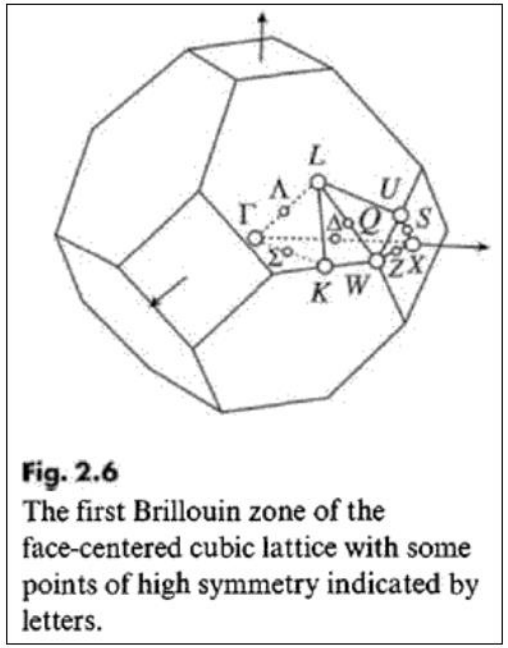


Fig. 2.6
The first Brillouin zone of the face-centered cubic lattice with some points of high symmetry indicated by letters.

Image from Ref. [2] pg.30

Acoustic Frequency Band Structure

- Example 2D Lattice of Cylinders, Ref. [3]
  - Solid Cylinders in Solid Matrix
  - Transverse Waves Considered
    (Particle Displacements Out of Plane)
- Acoustic Wave Equation Governs System
  - $\rho \frac{\partial^2 u}{\partial t^2} = \nabla_t \cdot (\rho c_t^2 \nabla_t u)$
- Solution Using Bloch's Theorem
  - $u(\vec{r}, t) = e^{i(\vec{k}\cdot\vec{r} - \omega t)} \sum_{\vec{G}} u_{\vec{k}}(\vec{G})\, e^{i\vec{G}\cdot\vec{r}}$
- **Frequency Band Gaps Exist – Forbidden Frequencies in Gaps**

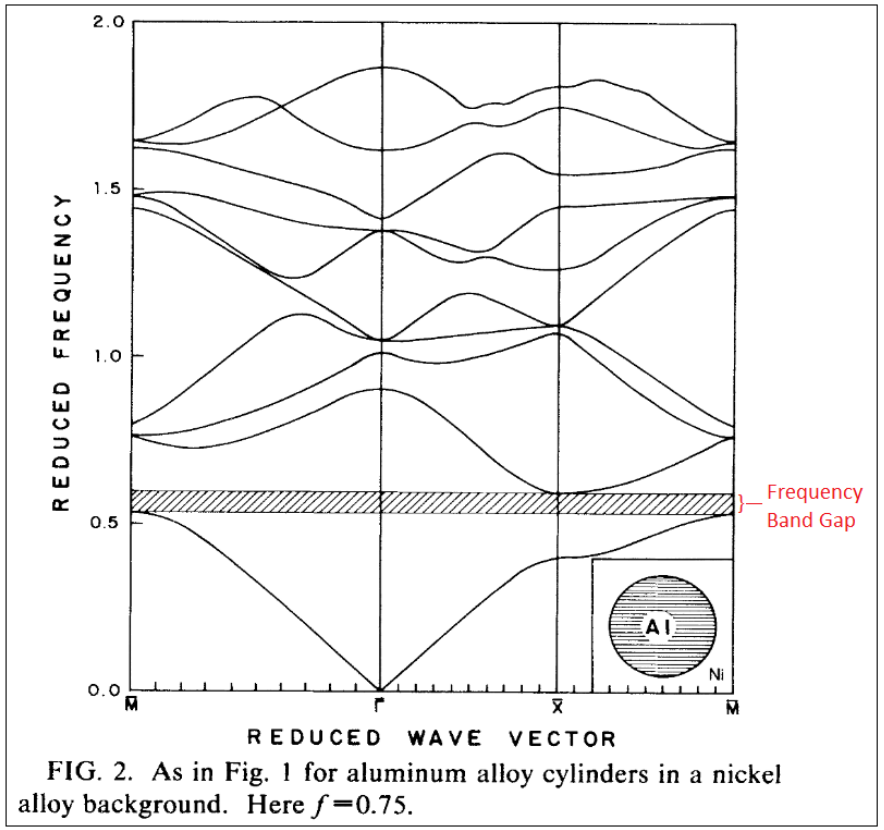


FIG. 2. As in Fig. 1 for aluminum alloy cylinders in a nickel alloy background. Here $f = 0.75$.

Image from Ref. [3] pg. 3

Quantum Hall Effect (QHE)

- Electrons in Magnetic Field
  - Circular Motions
  - Quantized Landau (Energy) Levels
- Quantized Conductivity & Resistance
- Gapless Edge States
  - Unidirectional Edge Current
  - Edge State Bridges Energy Bands
- Brillouin Zone Topology
  - Ref. [5], Chern Number: $n = \sum_m \frac{1}{2\pi} \int \mathcal{F}_m d^2\vec{k}$
  - Hall Conductivity $\propto n$
  - Non-Trivial Topology, $n \neq 0$
  - **Bulk-Boundary Correspondence, Ref. [5]**

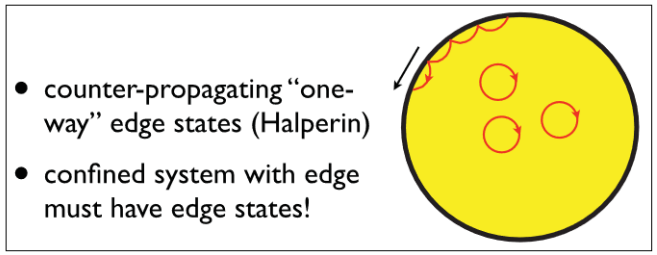


Image from Ref. [4] pg.2

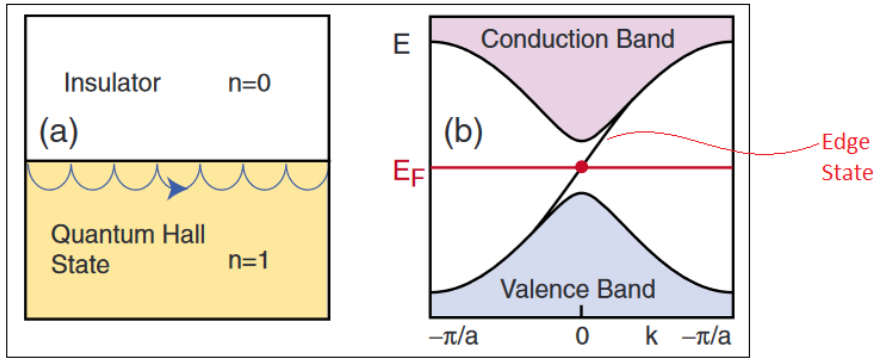
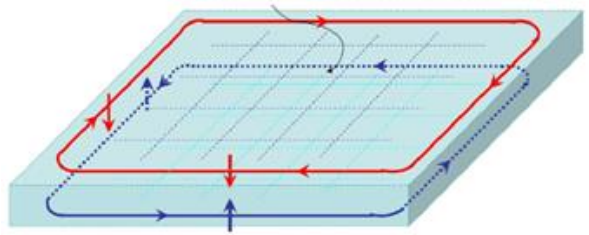


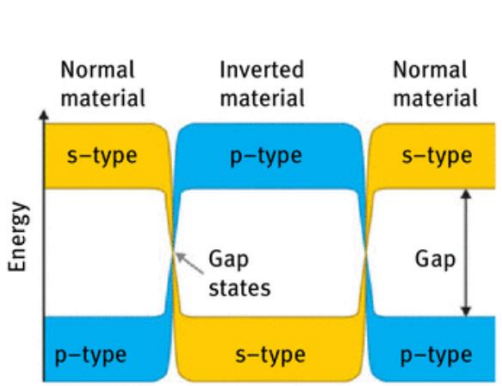
Image from Ref. [5] pg.4

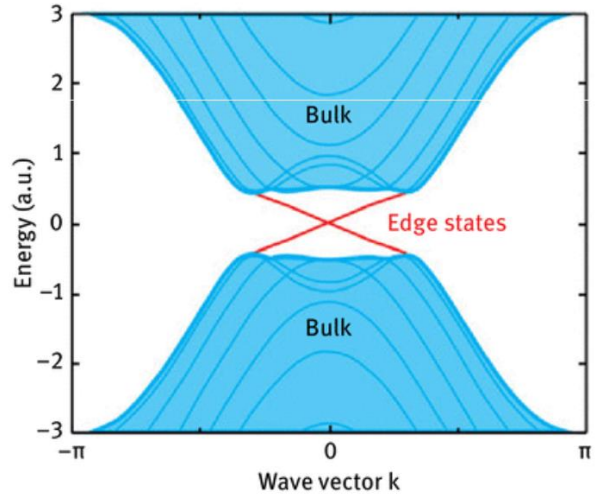
Quantum Spin Hall Effect (QSHE)

- Spin-Orbit Coupling
  - Similarity with Magnetic Field Using Electric Fields, Ref. [6] example $\vec{E}(r,\theta,z) = \vec{f}(r)$
  - HgTe/CdTe Semi-Conductor Quantum Well, Ref. [7]
- Pairs of Oppositely Traveling Particles on Edge with Opposing Spins
- Symmetric Gapless Edge States
  - Time-Reversal Invariant
- Spin Chern Number

Image from Ref. [6] pg. 3



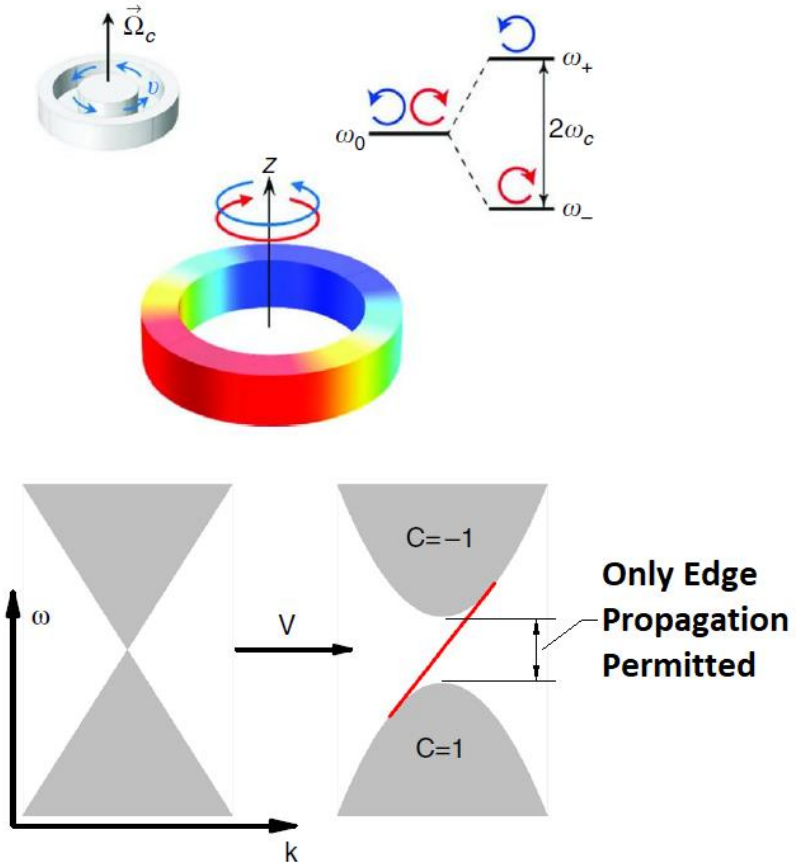




Images from Ref. [7] pg. 250

Acoustic Edge States and Topological Acoustics

- Edge States Similar to Quantum Hall and Spin Hall Materials are Possible for Sonic/Phononic Crystals
- Quantum Hall Analogue
  - Acoustic Non-Reciprocity and Time-Reversal Symmetry Breaking Required, Ref. [9]
  - Circulating Flow to Simulate Magnetic Field
- Quantum Spin Hall Analogue
  - Pseudospin Required
- **Waveguide Application – Edge Paths**
  - Input Pressure with Frequency in Band Gap
  - Topologically Protected Edge States Carry the Wave



Images from Ref. [9] pg. 3

Protection of Edge States

- Resistance Against Lattice Defects
  - Protected Against Backscattering
- No Open Bulk States
- Time-Reversal Symmetry Protection for QSHE
- Chern Number Independent of Material Properties

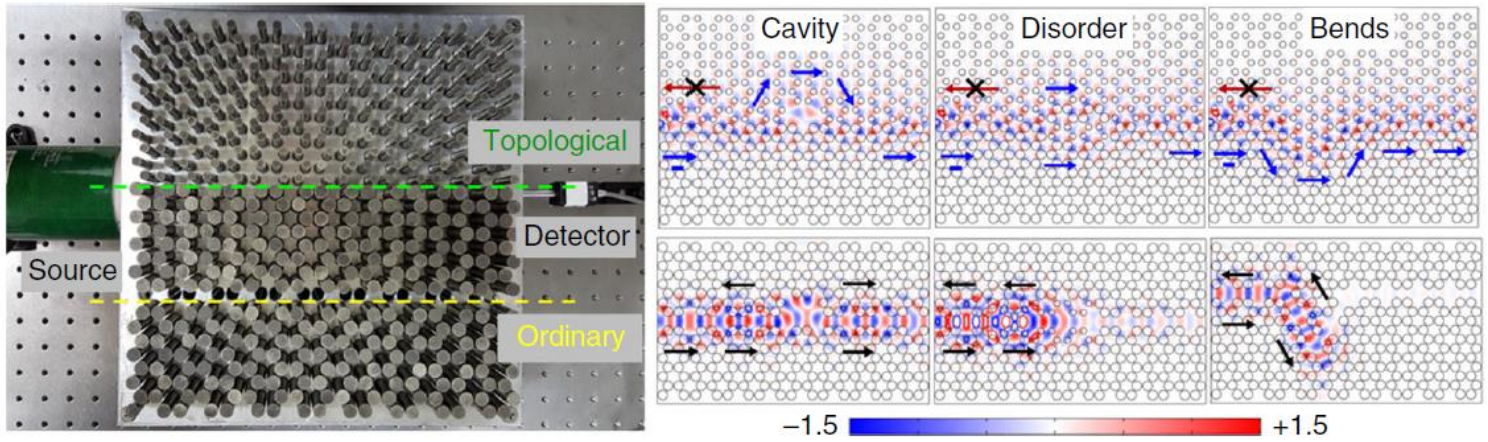
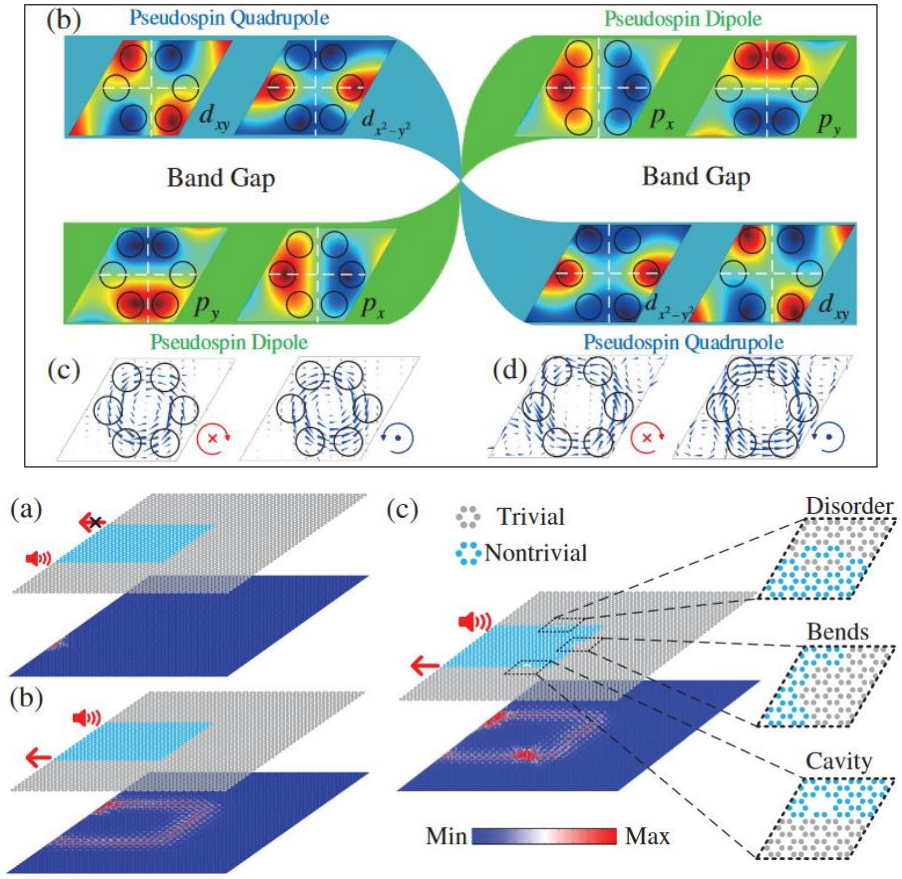


Image from Ref. [9] pg.5

Acoustic Topological Insulator Studies

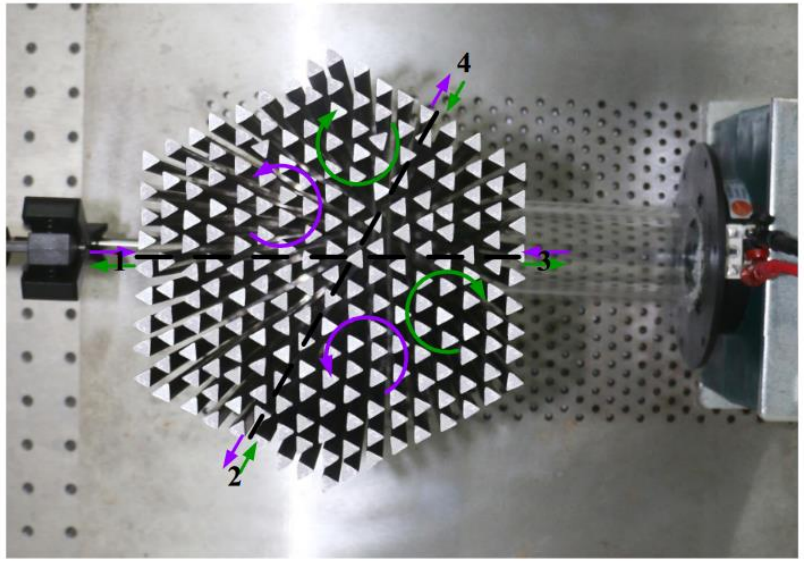
- Pseudospins in Unsymmetric Lattice, Ref. [10] & [11]
  - Topological Phase Transition Between
  - Circular Cylinder Rods Spacing in Hexagon Pattern to Topological Phase, Ref. [10]
- Simulations and Tests Verify Edge Propagation and Resistance to Lattice Defects

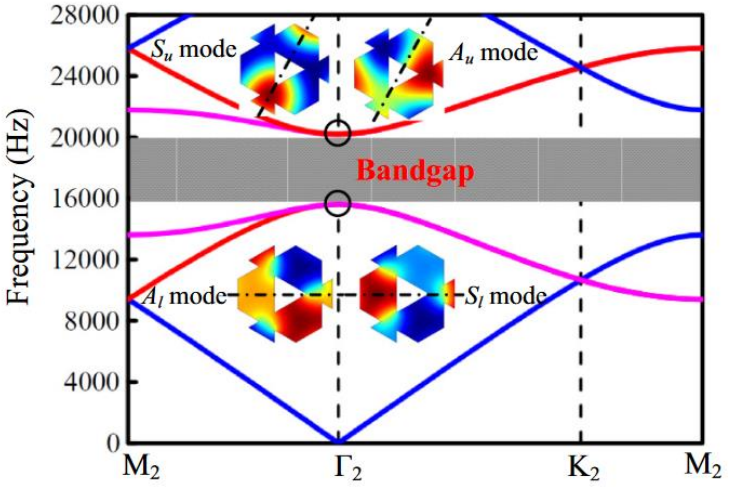


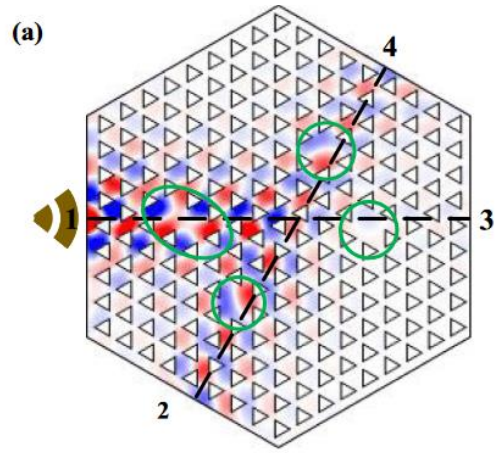
Images from Ref. [10] pg.3 & 5

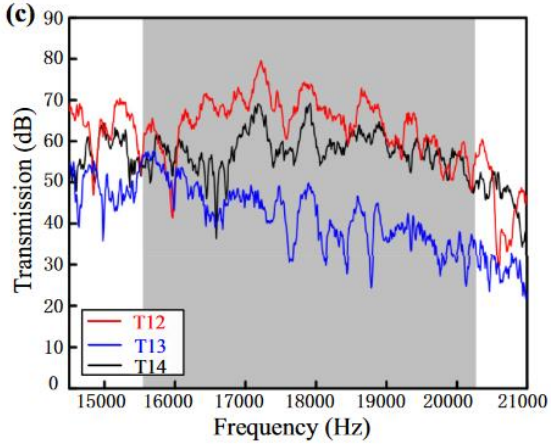
Acoustic Topological Insulator Studies (Continued)

- Ref. [11] Test Data and Equipment, Images from pgs. 2, 4, & 5
- Rotatable Triangular Prisms to Adjust Frequency Band Structure and Topological Phase



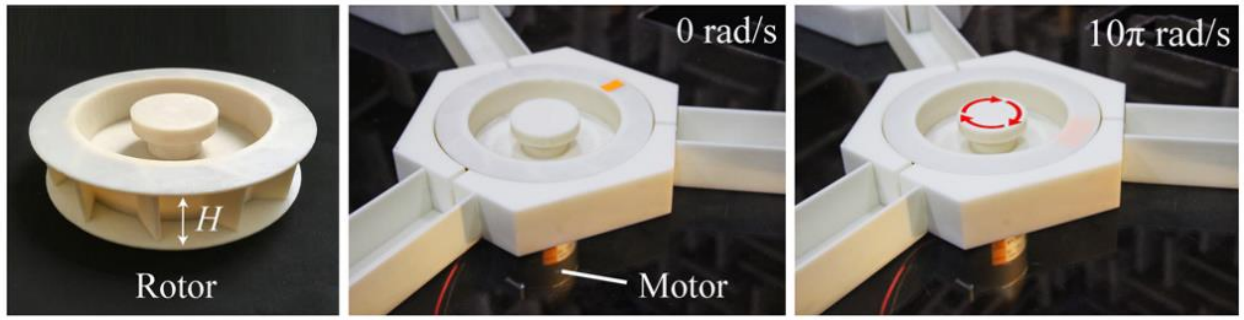




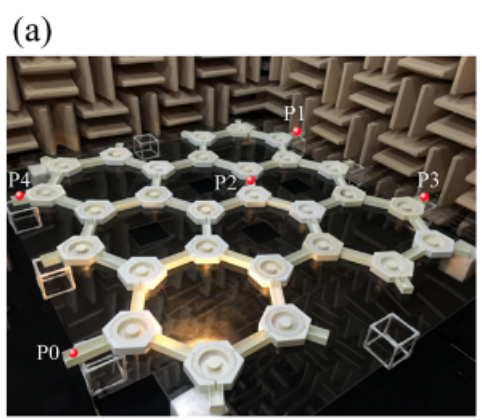


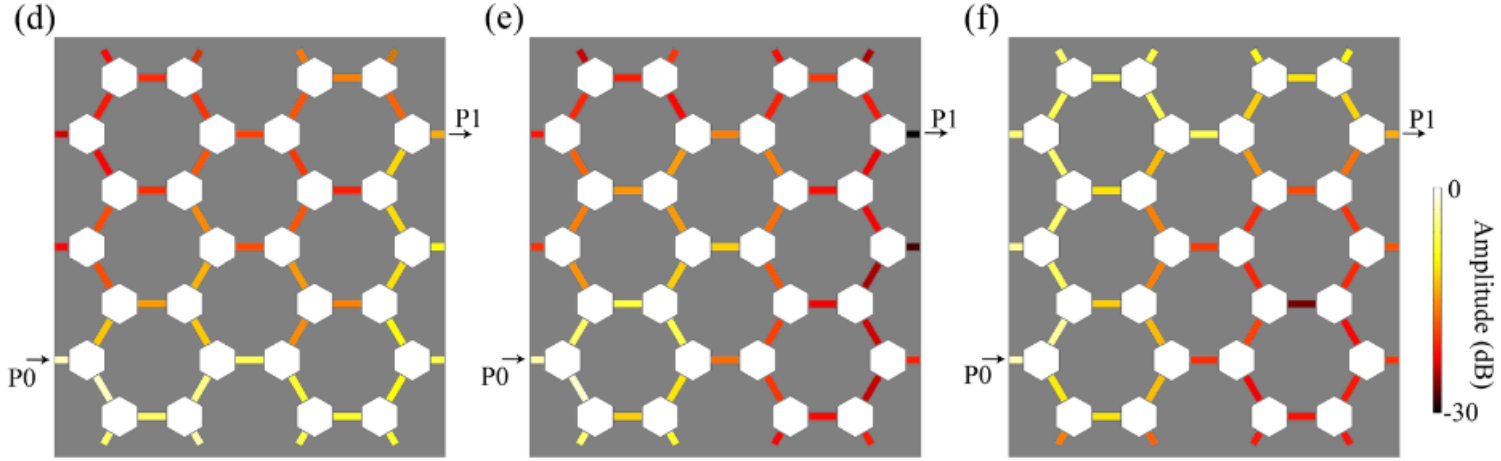
Acoustic Chern Insulator Test

- Test Results and Theory in Ref. [12]
- Active Motors Imparting Circulating Airflow
  - Simulates Effect of a Magnetic Field
- Edge States Observed with Resistance to Lattice Defects



Images from Ref. [12] pgs. 2 & 4





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