Phonon Echos

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Phonons

- Atoms vibrate, even at 0K.
- Phonon is a *quanta* of lattice vibrational energy.
- Single vibration → Spring (SHO)
- Monatomic lattice → Atoms connected by springs
- Diatomic lattice → Same, but different masses.

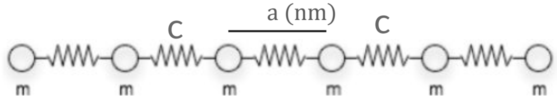
- What are the vibrational modes?

Monatomic lattice:

$$\omega(k) = \sqrt{\frac{4C}{m}} | \sin\left(\frac{ka}{2}\right) |$$

Diatomic lattice:

$$\omega^2(k) = C(m_1^{-1} + m_2^{-1}) \pm C\sqrt{(m_1^{-1} + m_2^{-1})^2 - 4\sin^2 ka\,(m_1 m_2)^{-1}}$$



$$F = ma$$

$$-kx_1 + k(x_2 - x_1) = m_1\ddot{x}_1$$

$$-k(x_2 - x_1) - kx_2 = m_2\ddot{x}_2$$

$$\ldots$$

$$x_1 = a_1 cos(\omega_0 t - \phi_1) + a_2 \cos(\sqrt{3}\omega_0 t - \phi_2)$$

$$x_2 = a_1 cos(\omega_0 t - \phi_1) + a_2 \cos(\sqrt{3}\omega_0 t - \phi_2)$$

$$x_1 = A_1 \cos(\omega t - \phi_1), x_2 = A_2 \cos(\omega t - \phi_2)$$

Multiple roots:

$$\hat{\omega} = \frac{\omega}{\omega_o}$$

$$\hat{\omega} = 1, A_1 = A_2 \qquad \text{(same direction)}$$

or

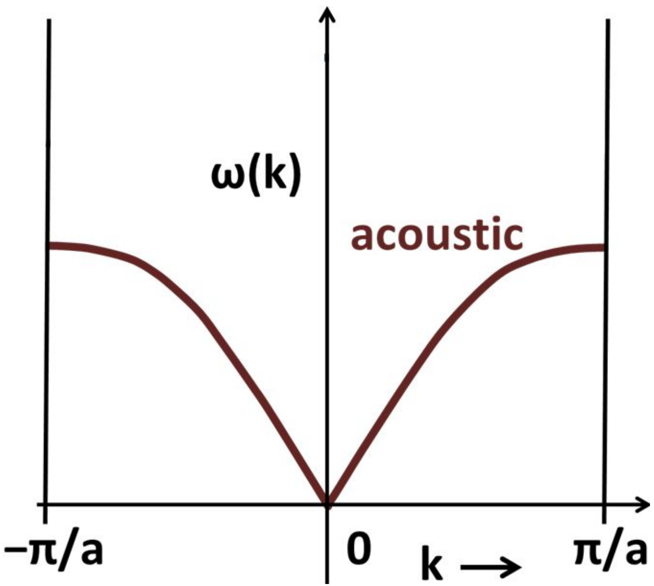
$$\hat{\omega} = \sqrt{3}, A_1 = -A_2 \quad \text{(opposite direction)}$$

Dispersion Relationship

- Frequency as a function of wave vector.
- One-dimensional lattice:

$$\omega = \sqrt{\frac{4k}{m}} \left| \sin\left(\frac{ka}{2}\right) \right|$$

- Notice the shape:
  - k=0, ω=0
  - increases in linear fashion
- Meaning atoms move in-phase with each other.
- Like a longitudinal wave, or sound wave.
  - Thus called the acoustic band

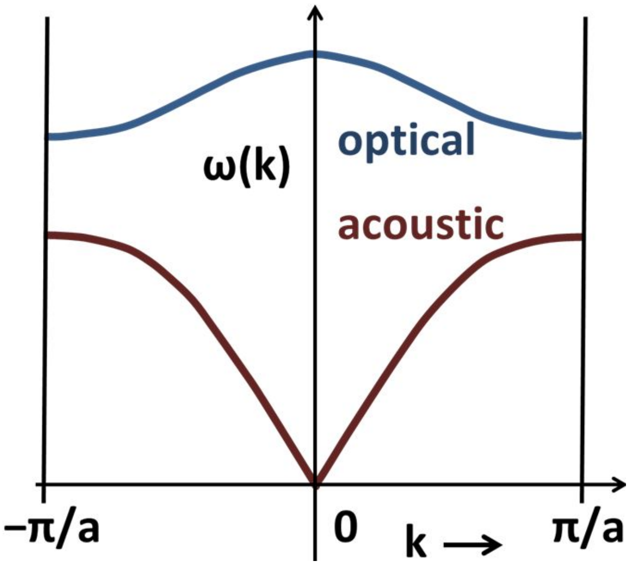


Dispersion Relationship

- Diatomic atom chain:

$$\omega^2(k) = C(m_1^{-1} + m_2^{-1}) \pm C\sqrt{(m_1^{-1} + m_2^{-1})^2 - 4\sin^2 ka\,(m_1 m_2)^{-1}}$$

- Optical Pattern:
    - k=0, $\omega_0(k) = \sqrt{2C\,(m_1^{-1} + m_2^{-1})}$
    - Doesn't change much with k.
    - You get higher frequency interactions from what you would expect from long wavelength interactions.



Phonon Gas Model

- Modes combine to form wave packets:

$$\psi = \cos(k_1 x - \omega_1 t) + \cos(k_2 x - \omega_2 t)$$

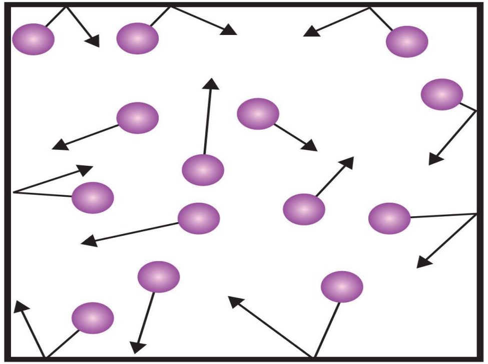
$$k_i = k - \Delta k, \quad w_i = \omega_i - \Delta\omega_i$$

$$\psi = 2cos(kx - \omega t)\cos(\Delta kx - \Delta\omega t)$$

$$\psi = 2[\text{inner wave}][\text{traveling wave}]$$

- Quantization
  - Harmonic oscillator: $E_n = \hbar\omega(n + \frac{1}{2})$
  - n = number of modes
  - Carrying energy ħω, Boson like
- Phonon: a quantum of lattice vibrational energy

Phonon Gas Model



Quasi particle

Phonon Echos

- Transducer inputs signal:

$$\text{Primary pulse, } \tau = 0: \quad \psi = a\exp(i\omega t)$$
$$\text{Secondary pulse:} \quad \psi = b\exp(i\omega(t-\tau))$$

- Linear coupling:
  - For a system with many eigen frequencies, echo effects go to zero due to phase cancelation.

$$\psi' = \sum_{\omega} a\exp(i\omega t) + b\exp(i\omega(t-\tau))$$

- Non-linear coupling
  - Echo effect at t = 2τ.

$$\psi' = \sum_{\omega} a^* \exp(i\omega t) b^2 \exp[-2i\omega(t-\tau)]$$
$$= \sum_{\omega} a^* b^2 \exp[-i\omega(t-2\tau)]$$

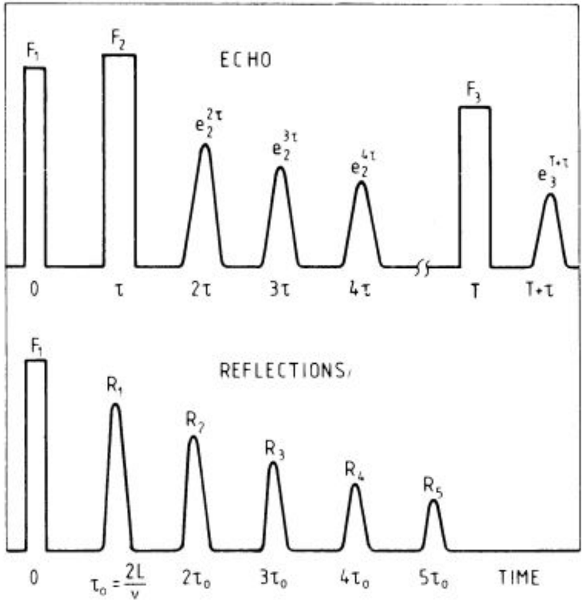
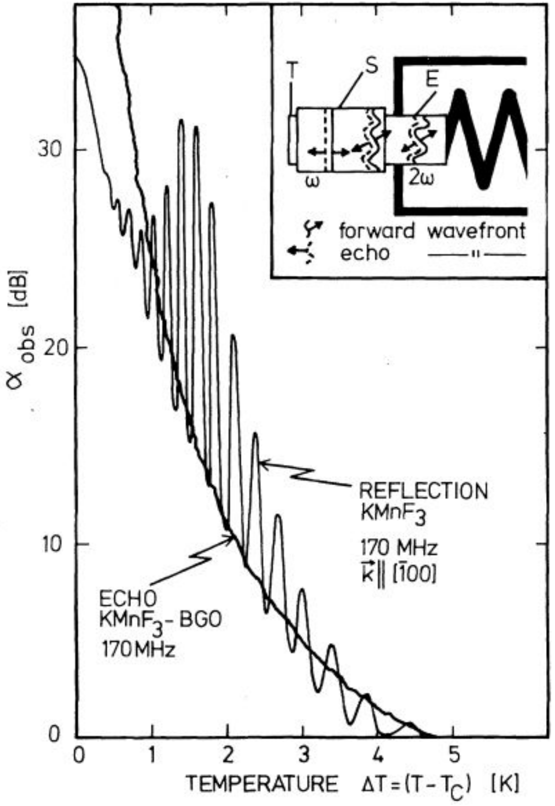


Fig. 1. Comparison of oscilloscope traces in an echo experiment (upper trace) in a powder, and in a pulsed reflection experiment (lower trace) (Fossheim and Holt 1982).

Phase Transitions

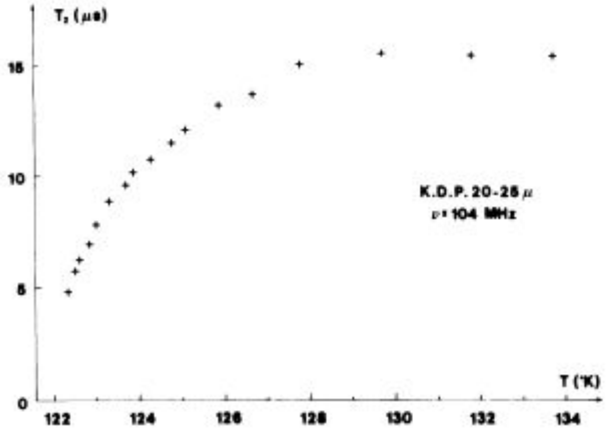
- KMnF3 has a structural phase transition at 178K.
- Effects the elastic constants → should be detectable.
- Ultrasonic problem:
  - Near the critical temperature, ultrasonic reflections add a lot of noise to the signal.
- Phonon echos produce a much cleaner signal.
  - Can extract relaxation time T_2.



Received signal vs temperature. Fossheim, K., and R. M. Holt. "Critical Dynamics of Sound in KMn F 3." Physical Review Letters 45.9 (1980): 730.

Relaxation Time (T_2)

- Potassium dihydrogen phosphate
- Structural phase change at 122K
- Using phonon echos, T_2 can be extracted.



T2 relaxation time vs temperature. Challis, L., ed. Phonon scattering in solids. Springer Science & Business Media, 2012.

Sources

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