Homework set 1 – chapters 1—5 in Kinsler

Solutions will be provided for these problems on Feb 1. Some of these are from Kinsler.
You can try additional problems in Kinsler if you wish – answers for odd-numbered problems are given in an Appendix. (Don't worry – most of these are harder than the final exam probs will be!)

1. Given the two complex numbers $\mathbf{A} = A\exp[j(\omega t + \theta)]$ and $\mathbf{B} = B\exp[j(\omega t + \phi)]$, find (*a*) the real part of **AB**, (*b*) the real part of **A**/**B**, (*c*) the real part of **A** times the real part of **B**, (*d*) the phase of **AB**, and (*e*) the phase of **A**/**B**.

2. A mass of 0.5 kg hangs on a spring. When an additional mass of 0.2 kg is attached to the spring, the spring stretches an additional 0.04 m. When the 0.2 kg mass is abruptly removed, the amplitude of the ensuing oscillations of the 0.5 kg mass is observed to decrease to 1/e of its initial value in 1.0 s. Compute values for R_m, ω_d, A, and ϕ.

3. An oscillator at rest experiences a force $F \sin \omega_0 t$ beginning at $t = 0$. If $\beta \ll \omega_0$, show that $x(t) \approx -(F/\omega_0 R_m)[1 - \exp(-\beta t)] \cos \omega_0 t$.

5. Using the spectral density $\mathbf{g}(w)$ of the force $F\exp(j\omega t)$ solve (1.15.13) for the resultant speed of an oscillator with mechanical impedance $\mathbf{Z}_m$.

6. A simple oscillator is suspended vertically with the spring unstretched. At time $t = 0$ the mass is released so that it is suddenly subjected to the force of gravity. Solve for the resulting displacement as a function of time.

8. Consider the waveform $y = 4\cos(3t - 2x)$ propagating on a string of linear density 0.1 g/cm, where y and x are in centimeters and t is in seconds. (*a*) What are the amplitude, phase speed, frequency, wavelength, and wave number? (*b*) What is the particle speed of the element at $x = 0$ at $t = 0$?

9. A mass of 0.2 kg is hung from a string of 0.05 kg mass and 1.0 m length. (*a*) What is the speed of transverse waves on the string? (Neglect the weight of the string in computing the tension.) (*b*) What are the frequencies of the fundamental and first overtone modes of transverse vibration of the string? (*c*) When the string is vibrating at its first overtone, what is the ratio of its displacement amplitude at the antinode to that of the mass?

10. A standing wave on a fixed, fixed string of length $L = 31.4$ cm and linear density 0.1 g/cm is given by $y = 2\sin(x/5)\cos(3t)$, where y and x are in centimeters and t is in seconds. (*a*) Find the phase speed, frequency, and wave number. (*b*) What is the amplitude of the particle displacement and speed at $x = L/2$ and $x = L/4$? (*c*) Find the energy density at these points. (*d*) How much energy is in the entire length of the string?

11. A steel bar of 0.0001 m^2 cross-sectional area and 0.25 m length is free to move at $x = 0$ and is loaded with 0.15 kg at $x = 0.25$ m. (*a*) Compute the fundamental frequency of longitudinal vibrations of the above mass-loaded bar. (*b*) Determine the position at which the bar may be clamped to cause the least interference with its fundamental mode of vibration. (*c*) When this bar is vibrating in its fundamental mode, what is the ratio of the displacement amplitude of the free end to that of the mass-loaded end? (*d*) What is the frequency of the first overtone of this bar?

12. Show that $v = \sqrt{\omega \kappa c}$ has the dimensions of a speed. For what frequency will the transverse vibrations of an aluminum rod of 0.01 m diameter have the same phase speed as that of longitudinal vibrations in the rod?

13. A rectangular membrane has width a and length b. If $b = 2a$, compute the ratio of each of the first four overtone frequencies relative to the fundamental frequency.

14. The diaphragm of a telephone receiver consists of a circular sheet of steel **4** cm in diameter and 0.02 cm thick. (*a*) If it is rigidly clamped at its rim, what is its fundamental frequency of vibration? What will be the effect on this frequency (*b*) of doubling the thickness of the diaphragm and (*c*) of doubling the diameter?

15. For a spherical wave $\mathbf{p} = (A/r)\cos(kr)\exp(j\omega t)$, find (*a*) the particle speed, (*b*) the specific acoustic impedance, (*c*) the instantaneous intensity, and (*d*) the intensity.

16. A plane sound wave in air of 100 Hz has a peak acoustic pressure amplitude of 2 Pa. (*a*) What is its intensity and its intensity level? (*b*) What is its peak particle displacement amplitude? (*c*) What is its peak particle speed amplitude? (*d*) What is its effective or rms pressure? (*e*) What is its sound pressure level *re* 20 μPa?

17. The receiving sensitivity level of a hydrophone is -80 dB *re* 1 V/μbar. (*a*) Express this level *re* 1 V/μPa. (*b*) What will be the (rms) output voltage if the pressure field is 80 dB *re* 1 μbar?