KEY - 2013

Homework 1

QSCI 482: Hypothesis Testing & Estimation for Ecologists & Resource Managers

1. Evaluate or simplify the following expressions, either numerically or symbolically as appropriate.

[a] $7\times2^2-3(2.2-1.7)$ = 26.5

[b] ln 7.3 = 2.0 or 1.99

[c] $e^{2.31}$ = 10.1 or 10.07

[d] $ln(4.5\ X^2)$ = $1.5 + 2\ln(X)$

[e] $e^{[\ln 5]}$ = 5

[f] if $f(X)=1.75\cdot X^{1.7}$, find $f(1.95)$ = 5.45

2. Often, a data set is described symbolically as X_1, X_2, …, X_n, representing the order in which the data were collected. Using the following data set: 14.0, 10.2, 12.8, 11.9, 15.2, 11.9, 14.7, (X_1, X_2, …, X_n, respectively) compute:

[a] $\sum_{i=1}^{3} 2X_i^2$. = 927.76

[b] $\sum_{i=4}^{7}(X_i - i)$ = 31.7

[c] $\sum_{i=1}^{3} X_i X_{i+1} - i$ = 419.68

[d] $\sum_{i=5}^{7}(X_i-2)/X_i$ = 2.56

[e] $\sum X$ = 90.7

[f] $\bar{X}$ = 12.96

[g] $\sum X^2$ = 1194.23

[h] $\left(\sum X\right)^2$ = 8226.49

[i] $\sum(X-\bar{X})^2$. = 19.02

3. The population of body weights for a small mammal is normally distributed with a population mean of 33.0 g [grams] and a population standard deviation of 6.5 g. (Consult Appendix Table B.2 in Zar.)

[a] What is the probability that an individual drawn at random from this population has a weight of at least 39 g?

[b] What is the probability that an individual drawn at random from this population will have a weight less than 39 g?

[c] What proportion of this population has a weight between 22 and 44 g?

4. Consider a certain population of insects. The body weights for this species are distributed as a normal random variable with a population mean (μ) of 127 mg [milligrams] and a population standard deviation (σ) of 22.1 mg.

[a] If a random sample of size 20 is drawn from this population, what is the probability that the sample average will be between 125 and 129 mg?

[b] How large a sample size would one have to take to end up with a standard error of the mean no greater than 2.5 mg?

3. [a] want $P(W \geq 39)$, where W = weight and $W \sim N(33, 42.25)$

$$P(W \geq 39)$$
$$= P\left(\frac{W-\mu}{\sigma} \geq \frac{39-\mu}{\sigma}\right)$$
$$= P\left(Z \geq \frac{39-33}{6.5}\right) = P(Z \geq 0.9231)$$

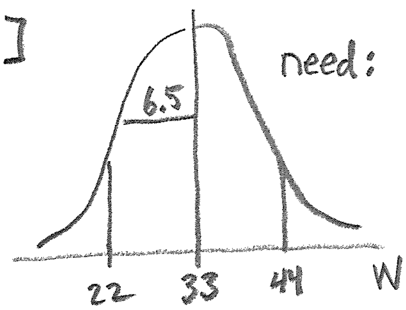
Zar Table B.2 gives $P(Z \geq 0.92) = 0.1788$

[b] $P(W < 39) = 1 - P(W \geq 39)$

From [a], $P(W \geq 39) = P(Z \geq 0.92) = 0.1788$

$\therefore \quad P(W < 39) = 1 - 0.1788 = 0.8212$

[c]

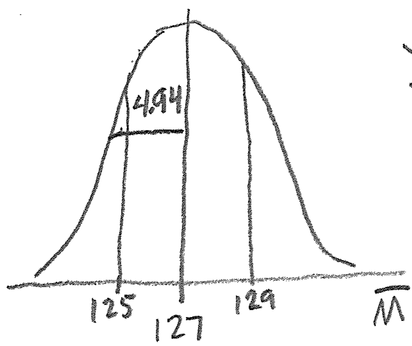


need: $P(22 \leq W \leq 44)$

$$= P(W \geq 22) - P(W \geq 44)$$
$$= P\left(Z \geq \frac{22-33}{6.5}\right) - P\left(Z \geq \frac{44-33}{6.5}\right)$$
$$= \underbrace{P(Z \geq -1.69)}_{1 - P(Z \leq -1.69)} - P(Z \geq 1.69)$$
$$= 1 - P(Z \geq 1.69) - P(Z \geq 1.69)$$
$$= 1 - 2P(Z \geq 1.69)$$
$$= 1 - 2(0.0455) = 0.909$$

4. [a] Given: Mass, $M \sim N(127, 488.41)$

$n = 20 \quad \therefore \quad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{22.1}{\sqrt{20}} = 4.94$



Desire: $P(125 \le \bar{M} \le 129) = ?$

$= P(\bar{M} \ge 125) - P(\bar{M} \ge 129)$

$= \underbrace{P(Z \ge -0.40)}_{1 - P(Z \le -0.40)} - P(Z \ge 0.40)$

$= 1 - P(Z \ge 0.40) - P(Z \ge 0.40)$

$= 0.3108$

[b] $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} \quad \therefore$

$\sqrt{n} \cdot \sigma_{\bar{X}} = \sigma$

$\sqrt{n} = \frac{\sigma}{\sigma_{\bar{X}}}$

$n = \frac{\sigma^2}{\sigma_{\bar{X}}^2} = \frac{488.41}{(2.5)^2} = 78.1$ so considering that only whole samples would be taken,

$n = 79.$