2013

QSCI 482 - Homework 2 - KEY

1. [a] $P(H > 120) = ?$

$$= P\left(\frac{H-100}{20} > \frac{120-100}{20}\right)$$
$$= P(Z > 1.0)$$
$$= 0.1587$$

Thus, about 15.9% of the pop'n is expected to be taller than 120 ft.

[b] $P(H \le h) = 0.30$ desire h

$P(Z \le -z) = P(Z > z)$ due to symmetry

so $P(Z > 0.524) = 0.30$

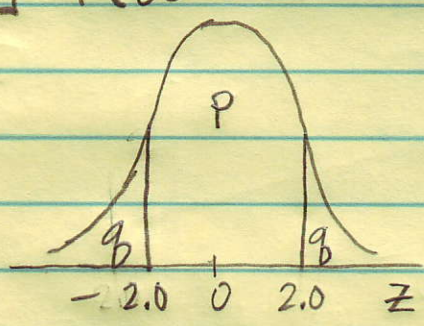
$\therefore$ $P(Z \le -0.524) = 0.30$

$= P\left(\frac{H-100}{20} \le -0.524\right) = 0.30$

$P(H \le 89.52) = 0.30$

Thus, since it is expected that 30% of trees will be shorter than 89.5 ft, 89.5 ft is the 30th percentile.

[c] $P(60 < H < 140) = ?$



$= P(-2.0 < Z < 2.0)$

$= 1 - 2q$ (see figure)

since $q = P(Z > 2.0) = 0.0228$

$= 1 - 2(0.0228)$

answer: 0.9544

[d] $\sigma_{\bar{X}} = \frac{20}{\sqrt{16}} = 5$; want $P(90 < \bar{X}_{16} < 110)$

$$= P\left(\frac{90-100}{5} < Z < \frac{110-100}{5}\right)$$
$$= P(-2.0 < Z < 2.0)$$

From the previous problem, answer: 0.9544

2013

QSCI 482 - Homework 2 - KEY

2. $P(\bar{X} - \sigma_{\bar{X}} \cdot Z_{0.025(1)} \le \mu \le \bar{X} + \sigma_{\bar{X}} \cdot Z_{0.025(1)}) = 0.95$

95% CI: $167 - \left(\sqrt{\frac{12}{30}}\right)1.96 \le \mu \le 167 + \left(\sqrt{\frac{12}{30}}\right) \cdot 1.96$

$165.8 \text{ lbs} \le \mu \le 168.2$

Unless a 5% chance occurred, a population mean that could have generated the sample is between 165.8 & 168.2

3 [a] $t_{0.001(1),30} = 3.385$

[b] $t_{0.03(2),11} = \omega_1 \cdot t_{.05(2),11} + \omega_2 \cdot t_{0.02(2)}$

$= 0.33(2.201) + 0.67(2.718)$

$= 2.547$

[c] $P(t_{10} > 2.764) = 0.01$

[d] $P(t_{66} < 1.295) = 1 - P(t_{66} > 1.295)$

$= 1 - 0.10$

$= 0.90$

[e] Let $P(t_{25} \ge 2.05) = q$

Since $P(t_{25} \ge 2.060) = 0.025$, and

$P(t_{25} \ge 2.485) = 0.01$, then

$0.01 \le q \le 0.025$

[f] $P(t_{11} < 1.5) = 1 - P(t_{11} \ge 1.5) = q$

Now, $P(t_{11} \ge 1.363) = 0.10$, and

$P(t_{11} \ge 1.796) = 0.05$

so $P(t_{11} \ge 1.5) = p$

$p = \omega_1(0.10) + \omega_2(0.05)$

$= \frac{1.796 - 1.5}{1.796 - 1.363}(0.10) + \frac{1.5 - 1.363}{1.796 - 1.363}(0.05)$

$= 0.084$

$\therefore P(t_{11} < 1.5) = 1 - 0.084 = 0.916$

2013

QSCI 482 – Homework 2 – KEY

4. $\bar{x} = 62$ chirps, $s = 12$ chirps, $n = 21$

[a] H_0 $\mu = \mu_0 = 68$ chirps/min
H_a: $\mu \neq \mu_0$

[b] Choose t-test because pop'n variance is unknown, data are continuous, normal dist'n is applicable, random & independent sampling assumed

$$t_{obs} = \frac{\bar{X} - \mu_0}{S_{\bar{X}}}. \quad \text{Here, } S_{\bar{X}} = \frac{12}{\sqrt{21}} = 2.619$$

$$t_{0.05(2),20} = 2.086$$

$$t_{obs} = \frac{62-68}{2.619} = -2.291$$

We reject the conjecture that the mean chirps/min for the pop'n from which the sample came is the same as that of the prevalent species and conclude we may have a new or sub-species here.

$$p\text{-value} = 0.05\left(\frac{2.528-2.291}{2.528-2.086}\right) + 0.02\left(\frac{2.291-2.086}{2.528-2.086}\right)$$

$$= 0.036$$

2013

QSCI 482 - Homework 2 - KEY

4. [c] $P(t_{20} \leq -2.086) = 0.025$

$$\Rightarrow P\left(\frac{\bar{X} - 68}{2.619} \leq -2.086\right) = 0.025$$

$$\Rightarrow P(\bar{X} \leq 62.54) = 0.025$$

Thus, a sample mean of 62.54 chirps/min would have given us a p-value equal to alpha, the significance level, chosen for the test.

Homework 2

QSCI 482: Hypothesis Testing & Estimation for Ecologists & Resource Managers

1. It is known that the heights of trees in a certain eucalyptus plantation are distributed as a normal random variable with mean $\mu = 100$ ft. and standard deviation $\sigma = 20$ ft..
   [a] What proportion of trees in this population would be larger than 120 ft?
   [b] What tree height represents the 30^{th} percentile of this distribution?
   [c] What proportion of trees have heights between 60 and 140 ft?
   [d] What chance is there that the mean of a sample of size 16 will lie between 90 and 110 ft?

2. Suppose weight measurements were taken on a sample of thirty male graduate students at the University of Illinois, exhibiting a mean of 167 lbs. Suppose also that it is known that this population has a population variance of 12 lbs^2. Find a range of plausible population means that could have generated this sample mean with 95% probability.

3. Consult Appendix Table B.3, Critical Values of the t-Distribution, in Zar.
   [a] Find $t_{0.001(1), 30}$
   [b] Find $t_{0.03(2), 11}$ (use interpolation to find the answer)
   [c] If df=10, find $P(t > 2.764)$.
   [d] If df=66, find $P(t < 1.295)$.
   [e] What is the probability of obtaining a value of t at least as large as 2.05, if df=25? Do not interpolate; just bracket the answer between two values.
   [f] What is the probability of obtaining a value of t smaller than 1.500, if df=11? Interpolate to find the answer this time.

4. While studying a particular species of cricket in a particular location, scientists found that the mating call of the males contained 62 “chirps” per minute under standard temperature with an observed standard deviation of 12 chirps per minute based on 21 observations. This cricket seems to be very similar in many other features to a well-documented prevalent species, but in some other ways it isn't, for example, because of the different mating call composition. Either a new or sub-species is suspected.

   [a] Suppose we want to compare these sample data to the well-documented, population mean of 68 chirps per minute for the prevalent species there. Write down the null and alternative hypotheses that would be tested assuming no fore-knowledge on whether the call of a new or sub-species would be longer or shorter.

   [b] Assuming (approximate) normality of the data (or at least regarding the behavior of the sample mean), do an appropriate test at the alpha = .05 level to see how the average mating call for this conjectured new or sub species compares with the well documented prevalent species value of 68 chirps per minute. Include the p-value associated with your test statistic. What do you conclude?

   [c] What value of the sample mean would have allowed us "just barely" to reject the null hypothesis? That is, find the value of the sample mean that would have put you right on the borderline between rejecting and not rejecting. (Assume still that n= 21 and s= 12.)