

**Instructions:** This test is closed book. However, you are allowed to use up to 6 pages of notes. You will have 50 minutes. The overall value of each problem, as well as the value for the components, are stated. You should use this information to allocate your time accordingly. For each problem, **show how you got your answer.** Partial credit will be given and the solution method will count more than the numerical values. Finally, please write your name on ALL extra sheets of paper you use. Good luck!

**Assembling Widgets (18).** The time to assemble a certain widget depends on the machine that does the assembly. Machine A has been designed to be a slow but reliable machine. With machine A, the assembly time is normally distributed with a mean of 5 minutes and a standard deviation of 0.5 minutes. Machine B, on the other hand, can be much faster. However, the assembly time can become quite long when the machine becomes dirty. As a result, the assembly time for machine B is exponentially distributed with a mean of 4 minutes.

As the plant engineer, your task is to decide which machine to purchase. Modeling of the plant has suggested that ideal operating conditions will occur when the manufacturing time is between 1 and 4 minutes. At the same time, the models suggest that serious flow problems will arise whenever assemblies require more than 8 minutes. You decide to explore the probable manufacturing times of the two machines, in order to make your decision.

POINTS

- 7 a. For each machine, what is the probability that the assembly time is greater than 8 minutes?
- 7 b. For each machine, what is the probability that the assembly time is between 1 and 4 min.?
- 4 c. Which machine should be purchased?

	MACHINE A	MACHINE B
	$T_A = \text{assembly time for A}$ $\sim N(5, 0.5)$	$T_B = \text{assembly time for B}$ $\sim \text{Exponential } (\lambda = \frac{1}{E(T)} = \frac{1}{4})$
(a)	$P(T_A > 8) = 1 - P(T_A < 8)$ $= 1 - P(Z < 6)$ $\approx 0$	$P(T_B > 8) = 1 - P(T_B < 8)$ $= 1 - [1 - e^{-\frac{1}{4}(8)}]$ $= 0.135$
(b)	$P(1 < T_A < 4) = P(T_A < 4) - P(T_A < 1)$ $= P(Z < -2) - P(Z < -8)$ $= 0.02275$	$P(1 < T_B < 4) = P(T_B < 4) - P(T_B < 1)$ $= [1 - e^{-\frac{1}{4}(4)}] - [1 - e^{-\frac{1}{4}(1)}]$ $= e^{-1} - e^{-\frac{1}{4}}$ $= 0.4109$

- (c) Machine A will have fewer times  $> 8$   
 Machine B will have more times in the ideal range.  
 Which machine? The answer depends on the cost of flow problems relative to benefits of ideal processing times. If clean machine B regularly  $\Rightarrow$  Maybe B.

change in the pasteurizing process for apple juice will be able to speed up the juice processing time. In the past, the processing time for a batch of juice has been 40 minutes. To explore whether the new process reduces pasteurization time, they pasteurize 6 batches of juice using the new process and collect the following pasteurization process times (in minutes): 36, 32, 35, 31, 38, 39. Please answer the following questions, to support the chemical engineering analysis:

POINTS

- 2 a. What is a point estimate for the ~~sample~~ mean?
- 7 b. What is a 95% confidence interval for the true mean?
- 4 c. What are the appropriate null and alternate hypotheses for the hypothesis test that will be able to support the chemical engineer's claims?
- 7 d. What is the result of the hypothesis test?
- 6 e. What is the p-value for this test?
- 6 f. If the true mean processing time for the new process is believed to be 36.8 minutes, what is the power of the test?
- 6 g. If the true mean processing time for the new process is believed to be 36.8 minutes, what sample size should be used to ensure that the power will be 0.90?
- 2 h. What assumption underpins this analysis?

Hint:  $\sum x_i^2 = 7421$ ,  $\sum x_i = 211$

(a)  $\bar{x} = 35.2$

(b) Because variance unknown, small sample  $\Rightarrow$  use t version  
 conf. interval:  $\bar{x} \pm t_{\alpha/2, n-1}(s)$ ,  $\alpha = 0.05$ ,  $n = 6$ ,  $s = 3.19$   
 $(31.85, 38.55)$

(c)  $H_0: \mu = 40$ ,  $H_a: \mu > 40$ .

(d) use T test  $t_0 = \frac{35.17 - 40}{3.19/\sqrt{6}} = -3.70$

reject if  $t_0 > t_{\alpha, n-1} \Rightarrow t_{0.05, 5} = 2.05 \Rightarrow$  REJECT

(e) PVALUE:  $P(T < t_0) = P(T < -3.70)$

$t_{0.01, 5} = 3.365$ ,  $t_{0.005, 5} = 4.032 \Rightarrow .005 < p < .01$

(f) B/c test conditions, use O.C curve

$d = \frac{M_0 - M}{\sigma} \approx \frac{40 - 36.8}{3.2} = 1$ , when  $n = 6$ ,  $B = 0.4$ , Power  $\approx 0.6$

(g) B/c t test conditions, must use Operating Char Curves

$d = \frac{M_0 - M}{\sigma} \approx \frac{40 - 36.8}{3.2} = 1$ , if want Power = 0.9,  $B = 0.1$

(h) NORMALITY of the juice processing times  $\Rightarrow n \approx 10$

particular curve in a widely traveled road, a team of civil engineers has decided to explore the speeds of cars entering the curve. Before they begin to collect data and test hypotheses, they perform some analysis to develop intuitions about the sample means that they might see. Highway records from previous testing suggest that the mean speed is 58 mph and the standard deviation of speed is 6 mph.

POINTS

- 6 a. If the civil engineering team collects 16 observations, what is the probability that the sample mean will be greater than 60 mph?
- 6 b. If the civil engineering team collects 100 observations, what is the probability that the sample mean will be greater than 60 mph?

$$X \approx \text{speed going into curve.} \\ \sim N(58, 5)$$

$$\bar{X} = \text{sample mean} \\ \sim N(58, 5/\sqrt{n}) \text{ when } n \text{ observations}$$

(a) when  $n = 16$

$$\begin{aligned} P(\bar{X} \geq 60) &= 1 - P(\bar{X} < 60) \\ &= 1 - P\left(Z < \frac{60 - 58}{6/\sqrt{16}}\right) \\ &= 1 - P(Z < 1.33) \\ &= 0.091759 \end{aligned}$$

(b) when  $n = 100$

$$\begin{aligned} P(\bar{X} > 60) &= 1 - P(\bar{X} < 60) \\ &= 1 - P\left(Z < \frac{60 - 58}{6/\sqrt{100}}\right) \\ &= 1 - P(Z < 3.33) \\ &= 0.000434 \end{aligned}$$

of cars entering an accident-prone curve, has collected the speed of 100 cars entering the curve. The sample mean of this data is 61.5 mph and the sample standard deviation is 7.2. The team thinks that both the mean speed entering the curve and the variability of car speed entering the curve could be contributing to the accident rate. Because of this, the civil engineering team would now like to explore two hypotheses - that the mean speed is greater than 60 mph and that the standard deviation of speed is greater than 6 mph.

POINTS

- 15 a. Is there evidence to demonstrate that the mean speed entering the curve is greater than 60 mph? (Use  $\alpha = 0.05$ )
- 15 b. Is there sufficient evidence to demonstrate that the standard deviation of speeds entering the curve is greater than 6 mph? (Use  $\alpha = 0.05$ )

Inventory:  $\bar{x} = 61.5, s = 7.2,$   
 $\mu_0 = 60, \sigma_0 = 6.$

(a) Test  $H_0: \mu = 60$  }  $\alpha = 0.05$   
 $H_1: \mu > 60$

Use Z b/c large sample  $Z_0 = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = 2.08.$

Reject if  $Z_0 > Z_\alpha$  and  $Z_{0.05} = 1.65$

$\therefore$  REJECT - There is evidence that true mean speed is greater than 60

(b) Test  $H_0: \sigma = 6$  |  $\alpha = 0.05$   
 $H_1: \sigma > 6.$

Use  $\chi^2$  test because test on std dev.

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{99(7.2)^2}{(6)^2} = 142.56$$

reject if  $\chi_0^2 > \chi_{0.05, 99}$ ,  $\chi_{0.05, 99} = 124.34.$

$\therefore$  REJECT. There is evidence that true standard deviation is greater than 6.